

Advanced Higher Physics

Practice Questions

(With solutions)

Contents

Rotational Motion and Astrophysics	5
Kinematic relationships	
Angular motion	
Centripetal force and acceleration	
Moment of inertia, torque and angular acceleration	
Angular momentum and rotational kinetic energy	
Gravitation	
Space and time	
Stellar physics	
Quanta and Waves	27
Quantum theory	
Particles from space	
Simple Harmonic Motion 1	
Simple Harmonic Motion 2	
Waves 1	
Waves 2	
Interference – division of amplitude 1	
Interference – division of amplitude 2	
Interference – division of wavefront 1	
Interference – division of wavefront 2	
Polarisation 1	
Polarisation 2	
Electromagnetism	52
Coulomb’s inverse square law and electric field strength	
Coulomb's Inverse Square Law	
Electric fields and electrostatic potential	

Electric Field Strength
Electrostatic Potential
Charges in motion 1
Charges in Motion 2
Electromagnetism
Force on a Conductor
Motion in a magnetic field
Charged Particles in Magnetic Fields
Capacitors in d.c. circuits
Capacitors in a.c. circuits
Self-inductance 1
Self-inductance 2
Inductors and a.c.
Electromagnetic radiation

Answers

95

DATA SHEET
COMMON PHYSICAL QUANTITIES

Quantity	Symbol	Value	Quantity	Symbol	Value
Gravitational acceleration on Earth	g	9.8 m s^{-2}	Mass of electron	m_e	$9.11 \times 10^{-31} \text{ kg}$
Radius of Earth	R_E	$6.4 \times 10^6 \text{ m}$	Charge on electron	e	$-1.60 \times 10^{-19} \text{ C}$
Mass of Earth	M_E	$6.0 \times 10^{24} \text{ kg}$	Mass of neutron	m_n	$1.675 \times 10^{-27} \text{ kg}$
Mass of Moon	M_M	$7.3 \times 10^{22} \text{ kg}$	Mass of proton	m_p	$1.673 \times 10^{-27} \text{ kg}$
Radius of Moon	R_M	$1.7 \times 10^6 \text{ m}$	Mass of alpha particle	m_α	$6.645 \times 10^{-27} \text{ kg}$
Mean Radius of Moon Orbit		$3.84 \times 10^8 \text{ m}$	Charge on alpha particle		$3.20 \times 10^{-19} \text{ C}$
Solar radius		$6.955 \times 10^8 \text{ m}$	Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Mass of Sun		$2.0 \times 10^{30} \text{ kg}$	Permittivity of free space	ϵ_0	$8.85 \times 10^{-12} \text{ F m}^{-1}$
1 AU		$1.5 \times 10^{11} \text{ m}$	Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$	Speed of light in vacuum	c	$3.0 \times 10^8 \text{ m s}^{-1}$
Universal constant of gravitation	G	$6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$	Speed of sound in air	v	$3.4 \times 10^2 \text{ m s}^{-1}$

REFRACTIVE INDICES

The refractive indices refer to sodium light of wavelength 589 nm and to substances at a temperature of 273 K.

Substance	Refractive index	Substance	Refractive index
Diamond	2.42	Glycerol	1.47
Glass	1.51	Water	1.33
Ice	1.31	Air	1.00
Perspex	1.49	Magnesium Fluoride	1.38

SPECTRAL LINES

Element	Wavelength/nm	Colour	Element	Wavelength/nm	Colour
Hydrogen	656	Red	Cadmium	644	Red
	486	Blue-green		509	Green
	434	Blue-violet		480	Blue
	410	Violet	<i>Lasers</i>		
	397	Ultraviolet	<i>Element</i>	<i>Wavelength/nm</i>	<i>Colour</i>
389	Ultraviolet	Carbon dioxide	9550 } 10590 }	Infrared	
Sodium	589	Yellow	Helium-neon	633	Red

PROPERTIES OF SELECTED MATERIALS

Substance	Density/ kg m^{-3}	Melting Point/ K	Boiling Point/K	Specific Heat Capacity/ $\text{J kg}^{-1} \text{ K}^{-1}$	Specific Latent Heat of Fusion/ J kg^{-1}	Specific Latent Heat of Vaporisation/ J kg^{-1}
Aluminium	2.70×10^3	933	2623	9.02×10^2	3.95×10^5
Copper	8.96×10^3	1357	2853	3.86×10^2	2.05×10^5
Glass	2.60×10^3	1400	6.70×10^2
Ice	9.20×10^2	273	2.10×10^3	3.34×10^5
Glycerol	1.26×10^3	291	563	2.43×10^3	1.81×10^5	8.30×10^5
Methanol	7.91×10^2	175	338	2.52×10^3	9.9×10^4	1.12×10^6
Sea Water	1.02×10^3	264	377	3.93×10^3
Water	1.00×10^3	273	373	4.19×10^3	3.34×10^5	2.26×10^6
Air	1.29
Hydrogen	9.0×10^{-2}	14	20	1.43×10^4	4.50×10^5
Nitrogen	1.25	63	77	1.04×10^3	2.00×10^5
Oxygen	1.43	55	90	9.18×10^2	2.40×10^4

The gas densities refer to a temperature of 273 K and a pressure of $1.01 \times 10^5 \text{ Pa}$.

Rotational Motion and Astrophysics

Kinematic relationships

1. The displacement, s , in metres, of an object after time t in seconds, is given by $s = 90t - 4t^2$.
 - (a) Using a calculus method, find an expression for the object's velocity.
 - (b) At what time will the velocity be zero?
 - (c) Show that the acceleration is constant and state its value.

2. The displacement, s , in metres of a 3 kg mass is given by $s = 8 - 10t + t^2$, where t is the time in seconds.
 - (a) Calculate the object's velocity after:
 - (i) 2 s
 - (ii) 5 s
 - (iii) 8 s.
 - (b) Calculate the unbalanced force acting on the object after 4 s.
 - (c) Comment on the unbalanced force acting on the object during its journey.

3. The displacement, s , of a car is given by the expression $s = 5t + t^2$ metres, where t is in seconds.

Calculate:

 - (a) the velocity of the car when the timing started
 - (b) the velocity of the car after 3 seconds
 - (c) the acceleration of the car
 - (d) the time taken by the car to travel 6 m after the timing started.

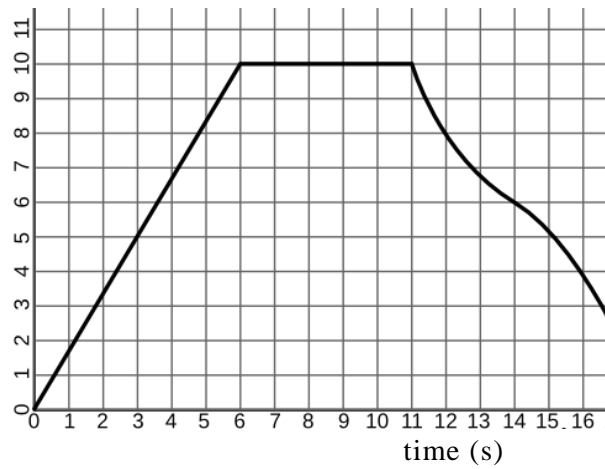
4. The displacement, s , of an object is given by the expression $s = 3t^3 + 5t$ metres, where t is in seconds.
 - (a) Calculate the displacement, speed and acceleration of the object after 3 seconds.
 - (b) Explain why the unbalanced force on the object is not constant.

5. An arrow is fired vertically in the air. The vertical displacement, s , is given by $s = 34.3t - 4.9t^2$ metres, where t is in seconds.
 - (a) Find an expression for the velocity of the arrow.
 - (b) Calculate the acceleration of the arrow.
 - (c) Calculate the initial velocity of the arrow.
 - (d) Calculate the maximum height reached by the arrow.

6. The displacement, s , of an object is given by $s = 12 + 15t^2 - 25t^4$ metres, where t is in seconds.
- Find expressions for the velocity and acceleration of the object.
 - Determine the object's initial:
 - displacement
 - velocity
 - acceleration.
 - At what times is the velocity of the object zero?
7. The displacement, s , of a rocket launched from the Earth's surface is given by $s = 2t^3 + 8t^2$ metres for $0 \leq t \leq 30$ seconds.
- Calculate the speed of the rocket after 15 seconds.
 - How far had the rocket travelled in 30 s?
 - Suggest a reason why the expression for displacement is only valid for the first 30 s.
8. A box with a constant acceleration of 4 m s^{-2} slides down a smooth slope. At time $t = 0$ the displacement of the box is 2 m and its velocity is 3 m s^{-1} .
- Use a calculus method to show that the velocity v of the box is given the expression $v = 4t + 3 \text{ m s}^{-1}$.
 - Show that the displacement of the box is given by $s = 2t^2 + 3t + 2$ metres.
9. The velocity, v , of a moving trolley is given by $v = 6t + 2 \text{ m s}^{-1}$. The displacement of the trolley is zero at time $t = 0$.
- Derive an expression for the displacement of the trolley.
 - Calculate the acceleration of the trolley.
 - State the velocity of the trolley at time $t = 0$.

10. The following graph shows the displacement of an object varying with time.

displacement (m)

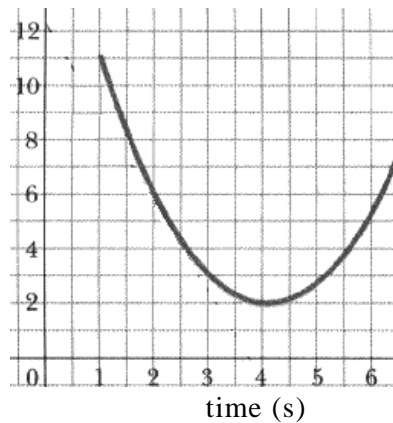


Calculate the velocity of the object at:

- (a) 3 s
- (b) 8 s
- (c) 12 s.

11. The following graph shows how the velocity of an object changes with time.

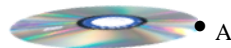
velocity (m s^{-1})



- (a) Calculate the acceleration of the object at 2 s.
- (b) At what time is the acceleration zero?
- (c) Estimate the distance travelled between 2 s and 5 s.

Angular motion

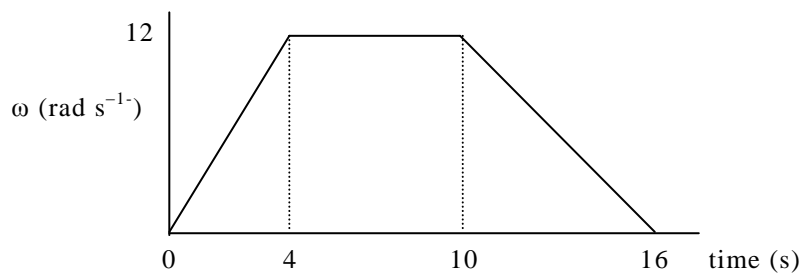
- Convert the following from degrees into radians:
 180° , 360° , 90° , 60° , 30° , 14° , 1°
- Convert the following from radians to degrees:
 π rad, 2π rad, $\frac{1}{2}\pi$ rad, 1 rad, 5 rad, 0.1 rad, 0.01 rad
- Calculate the angular velocity of each of the following:
 - A bicycle spoke turning through 5.8 rad in 3.6 s.
 - A playground roundabout rotating once every 4 s.
 - An electric drill bit rotating at 3000 revolutions per minute (rpm).
 - An electric drill bit rotating at 40 revolutions per second.
 - The second hand of an analogue watch.
 - The Moon orbiting the Earth with a period of 27.3 days.
 - The Earth spinning about its polar axis.
 - A rotating object whose angular displacement, θ , is given by $\theta = 5t + 4$ radians, where t is the time in seconds.
- A propeller rotates at 95 rpm.
 - Calculate the angular velocity of the propeller.
 - Each propeller blade has a length of 0.35 m.
Calculate the linear speed of the tip of a propeller.
- A CD of diameter 120 mm rotates inside a CD player.



The linear speed of point A on the circumference of the CD is 1.4 m s^{-1} .
Calculate the angular velocity of the CD:

- in rad s^{-1}
 - in rpm.
- A rotating disc accelerates uniformly from 1.5 rad s^{-1} to 7.2 rad s^{-1} in 4 s.
 - Calculate the angular acceleration of the disc.
 - Calculate the total angular displacement in this time.
 - How many revolutions does the disc make in this time?

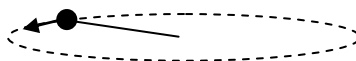
7. A washing machine drum slows down uniformly from 900 rpm to rest in 15 s.
- Calculate the angular acceleration of the drum.
 - How many revolutions does the drum make in this time?
8. A bicycle wheel rotating at 300 rpm makes 120 complete revolutions as it slows down uniformly and comes to rest.
- Calculate the angular acceleration of the wheel.
 - Calculate the time taken by the wheel to stop.
9. The graph shows how the angular velocity of a rotating drum varies with time.



- Calculate the initial and final angular acceleration of the drum.
- Calculate the total angular displacement of the drum.
- How many revolutions does the drum make in 16 s?

Centripetal force and acceleration

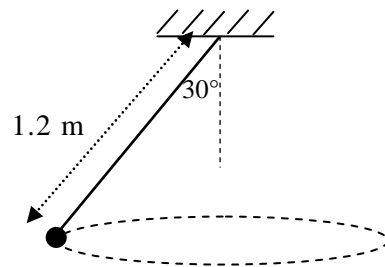
1. A mass of 150 g is attached to a string of length 1.2 m. The string is used to whirl the mass in a horizontal circle at two revolutions per second.
 - (a) Calculate the centripetal acceleration of the mass.
 - (b) Calculate the centripetal (central) force acting on the mass.
 - (c) The string has a breaking force of 56 N. Calculate the maximum angular velocity of the mass.
2. A mass of 0.50 kg is attached to a string of length 0.45 m and rotated in a horizontal circle. The mass has a linear (tangential) speed of 7.6 m s^{-1} .



Calculate the tension in the string.

3. A 3.0 kg mass attached to a string of length 0.75 m rotates in a vertical circle with a steady speed of 8.0 m s^{-1} .
 - (a) Calculate the tension in the string when the mass is at the top of the circle.
 - (b) Calculate the tension in the string when the mass is at the bottom of the circle.
 - (c) Calculate the minimum speed required for the mass to move in this vertical circle.
4. In a space flight simulator an astronaut is rotated horizontally at 20 rpm in a pod on the end of a radius arm of length 5.0 m. The mass of the astronaut is 75 kg.
 - (a) Calculate the central force on the astronaut.
 - (b) Show that this force is equivalent to a gravitational force of 2.2 g.
 - (c) Calculate the rotation rate in rpm that would give a 'simulated' gravity of 3 g.
5. A wet cloth of mass 50 g rotates at 1200 rpm in a spin-dryer drum of diameter 0.45 m. Calculate the central force acting on the cloth.

6. A small object of mass m revolves in a horizontal circle at a constant speed on the end of a string.



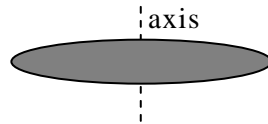
The string has a length of 1.2 m and makes an angle of 30° to the vertical as the mass rotates.

- (a) Name the two forces acting on the mass and draw a diagram showing the two forces acting on the mass.
- (b) Resolve the tension T in the string into a horizontal component and a vertical component.
- (c)
 - (i) Which component of the tension balances the weight of mass m ?
 - (ii) Write down an equation which describes this component.
- (d)
 - (i) Which component of the tension provides the central force to keep the mass moving in a circle?
 - (ii) Write down an equation using the central force and one of the components of the tension.
- (e) Calculate the radius of the circle using trigonometry.
- (f) Calculate the linear speed of the mass.
- (g) Calculate the period of the motion.

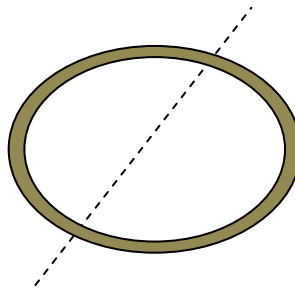
Moment of inertia, torque and angular acceleration

1. Calculate the moment of inertia of:

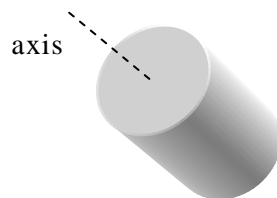
- (a) a disc of mass 2.3 kg and radius 0.75 m rotating about this axis



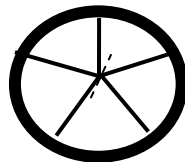
- (b) a rod of mass 0.45 kg and length 0.6 m rotating about its centre
(c) a rod of mass 1.2 kg and length 0.95 m rotating about its end
(d) a sphere of mass 12 kg and radius 0.15 m about an axis through its centre
(e) a point mass of 8.5×10^{-2} kg rotating 7.5×10^{-2} m from the rotation axis
(f) a metal ring of mass 2.1 kg and radius 0.16 m rotating about its central axis of symmetry



- (g) a solid cylinder of mass 4.5 kg and diameter 0.48 m rotating about the axis shown.

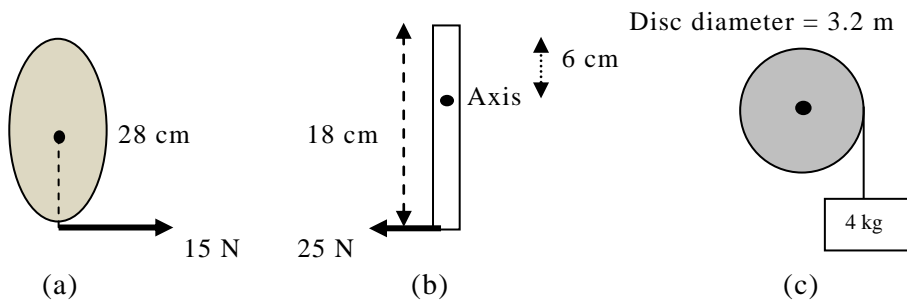


2. A wheel can be represented by a rim and five spokes.

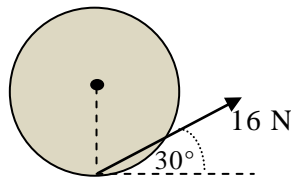


The rim has a mass of 1.5 kg and each spoke has a length of 0.55 m and mass of 0.32 kg. Calculate the moment of inertia of the wheel, assuming rotation about its axle (dotted line).

3. Calculate the torque applied about the axis in each of the following (the axis of rotation is represented by ●):



4. Calculate the torque applied by the 16 N force to the disc of radius 120 mm rotating about the axis represented by ●.



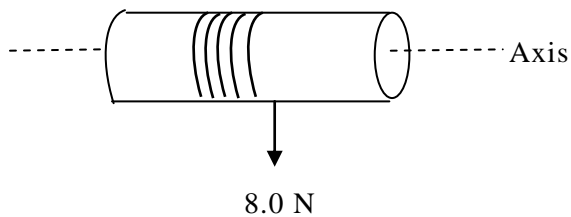
5. An engineer using a spanner of length 22 cm applies a torque of 18 N m to a nut.



Calculate the force exerted by the engineer.

6. A flywheel has a moment of inertia of 1.2 kg m^2 and is acted on by an unbalanced torque of 0.80 N m .
- Calculate the angular acceleration of the flywheel.
 - The unbalanced torque acts for 5 s and the flywheel starts from rest. Calculate:
 - the angular velocity at the end of the 5 s
 - the number of revolutions made in the 5 s.
7. A hoop of mass 0.25 kg and radius 0.20 m rotates about its central axis. Calculate the torque required to give the hoop an angular acceleration of 5.0 rad s^{-2} .

8. A solid drum has a moment of inertia of 2.0 kg m^2 and radius 0.50 m .
The drum rotates freely about its central axis at 10 rev s^{-1} .
A constant frictional force of 5.0 N is exerted tangentially to the rim of the drum.
Calculate:
- the time taken for the drum to come to rest
 - the number of revolutions made during the braking period
 - the heat generated during the braking.
9. A flywheel, with a moment of inertia of 1.5 kg m^2 , is driven by an electric motor which provides a driving torque of 7.7 N m . The flywheel rotates with a constant angular velocity of 52 rad s^{-1} .
- State the frictional torque acting on the flywheel. Give a reason for your answer.
 - The electric motor is now switched off.
Calculate the time taken for the flywheel to come to rest.
State any assumption you have made.
10. A cylindrical solid drum has a rope of length 5.0 m wound round it.
The rope is pulled with a constant force of 8.0 N and the drum is free to rotate about its central axis as shown.



The radius of the drum is 0.30 m and its moment of inertia about the axis is 0.40 kg m^2 .

- Calculate the torque applied to the drum.
- Calculate the angular acceleration of the drum, ignoring any frictional effects.
- Calculate the angular velocity of the drum just as the rope leaves the drum, assuming the drum starts from rest.

11. A bicycle wheel is mounted so that it can rotate horizontally as shown.



The wheel has a mass of 0.79 kg and radius of 0.45 m, and the masses of the spokes and axle are negligible.

- (a) Show that the moment of inertia of the wheel is 0.16 kg m^2 .
 - (b) A constant driving force of 20 N is applied tangentially to the rim of the wheel. Calculate the magnitude of the driving torque on the wheel.
 - (c) A constant frictional torque of 1.5 N m acts on the wheel. Calculate the angular acceleration of the wheel.
 - (d) After a period of 4 s, and assuming the wheel starts from rest, calculate:
 - (i) the total angular displacement of the wheel
 - (ii) the angular velocity of the wheel
 - (iii) the kinetic energy of the wheel.
 - (e) The driving force is removed after 4 s. Calculate the time taken for the wheel to come to rest.
12. A playground roundabout has a moment of inertia of 500 kg m^2 about its axis of rotation.

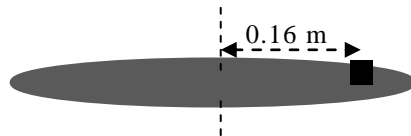


A constant torque of 200 N m is applied tangentially to the rim of the roundabout.

- (a) The angular acceleration of the roundabout is 0.35 rad s^{-2} . Show that the frictional torque acting on the roundabout is 25 N m.
- (b) A child of mass 50 kg sits on the roundabout at a distance of 1.25 m from the axis of rotation and the 200 N m torque is reapplied. Calculate the new angular acceleration of the roundabout.
- (c) The 200 N m torque in part (b) is applied for 3 s then removed.
 - (i) Calculate the maximum angular velocity of the roundabout and child.
 - (ii) The 200 N m torque is now removed. Find the time taken by the roundabout and child to come to rest.

Angular momentum and rotational kinetic energy

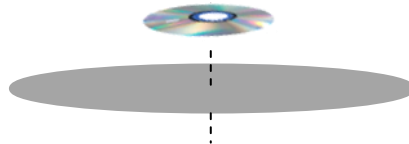
1. A bicycle wheel has a moment of inertia of 0.25 kg m^2 about its axle. The wheel rotates at 120 rpm. Calculate:
 - (a) the angular momentum of the wheel
 - (b) the rotational kinetic energy of the wheel.
2. A turntable of moment of inertia $5.8 \times 10^{-2} \text{ kg m}^2$ rotates freely at 3.5 rad s^{-1} with no external torques. A small mass of 0.18 kg falls vertically onto the turntable at a distance of 0.16 m from the axis of rotation.



Calculate the new angular speed of the turntable.

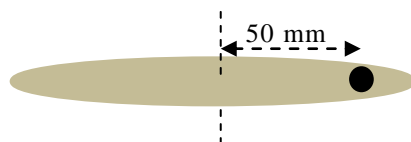
3. A turntable rotates freely at 40 rpm about its vertical axis. A small mass of 50 g falls vertically onto the turntable at a distance of 80 mm from the central axis. The rotation of the turntable is reduced to 33 rpm. Calculate the moment of inertia of the turntable.

A CD of mass 0.020 kg and diameter 120 mm is dropped onto a turntable rotating freely at 3.0 rad s^{-1} .



The turntable has a moment of inertia of $5.0 \times 10^{-4} \text{ kg m}^2$ about its rotation axis.

- (a) Calculate the angular speed of the turntable after the CD lands on it. Assume the CD is a uniform disc with no hole in the centre.
 - (b) Will your answer to part (a) be bigger, smaller or unchanged if the hole in the centre of the CD is taken into account? Explain your answer.
4. A turntable rotates freely at 100 rpm about its central axis. The moment of inertia of the turntable is $1.5 \times 10^{-4} \text{ kg m}^2$ about this axis. A mass of plasticine is dropped vertically onto the turntable and sticks at a distance of 50 mm from the centre of the turntable.



The turntable slows to 75 rpm after the plasticine lands on it.

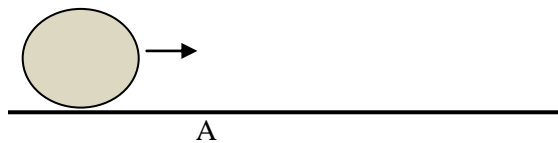
Calculate the mass of the plasticine.

5. An ice skater is spinning with an angular velocity of 3.0 rad s^{-1} with her arms outstretched.



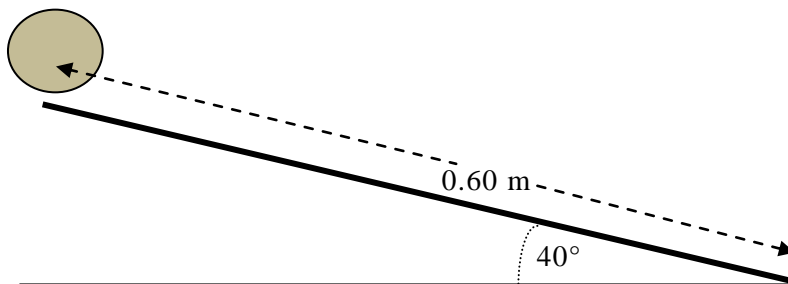
The skater draws in her arms and her angular velocity increases to 5.0 rad s^{-1} .

- (a) Explain why the angular velocity increases.
(b) When the skater's arms are outstretched her moment of inertia about the spin axis is 4.8 kg m^2 .
Calculate her moment of inertia when her arms are drawn in.
(c) Calculate the skater's change in rotational kinetic energy.
(d) Explain why there is a change in kinetic energy.
6. A solid sphere of mass 5.0 kg and radius 0.40 m rolls along a horizontal surface without slipping. The linear speed of the sphere as it passes point A is 1.2 m s^{-1} .



As the sphere passes point A calculate:

- (a) the linear kinetic energy of the sphere
(b) the angular velocity of the sphere
(c) the rotational kinetic energy of the sphere
(d) the total kinetic energy of the sphere.
7. A solid cylinder of mass 3.0 kg and radius 50 mm rolls down a slope without slipping.



The slope has a length of 0.60 m and is inclined at 40° to the horizontal.

- (a) Calculate the loss in gravitational potential energy as the cylinder rolls from the top to the bottom of the slope.
- (b) Calculate the linear speed of the cylinder as it reaches the bottom of the slope.

Gravitation

Astronomical data:

mass of Earth	=	6.0×10^{24} kg
radius of Earth	=	6.4×10^6 m
mean radius of Earth orbit	=	1.5×10^{11} m
mass of Moon	=	7.3×10^{22} kg
radius of Moon	=	1.7×10^6 m
mean radius of Moon orbit	=	3.84×10^8 m
mass of Mars	=	6.4×10^{23} kg
radius of Mars	=	3.4×10^6 m
mass of Sun	=	2.0×10^{30} kg

1. Calculate the gravitational force between two cars each of mass 1000 kg and parked 0.5 m apart.
2. Two large ships, each of mass 5.0×10^4 tonnes, are separated by a distance of 20 m. Show that the force of attraction between them is 417 N (1 tonne = 1000 kg).
3. Calculate the force of attraction between the Earth and the Sun.
4. The gravitational field strength at the surface of the Earth is 9.8 N kg^{-1} . Calculate the mass of the Earth.
5. Calculate the gravitational field strength on the surface of:
 - (a) Mars
 - (b) the Moon.
6. The gravitational field strength changes with altitude above sea level. Calculate the gravitational field strength at these locations:
 - (a) at the summit of Ben Nevis (height 1344 m)
 - (b) at the summit of Mount Everest (height 8848 m)
 - (c) on board an aircraft cruising at 12000 m
 - (d) on board the International Space Station orbiting at 350 km above the Earth.
7. A satellite of mass m orbits a planet of mass M and radius R .
 - (a) Show that the time for one complete orbit T (called the period of the satellite) is given by the expression
$$T^2 = \frac{4\pi^2}{GM} R^3$$
Is the time T dependent on the mass of the satellite?
8. A satellite orbits the Earth at a height of 250 km above the Earth's surface.
 - (a) Calculate the radius of the orbit of this satellite.
 - (b) Calculate the time taken by the satellite to make one orbit of the Earth.
9. A satellite orbits the Earth with a period of 95 min. Calculate the height of the satellite above the Earth's surface.

10. A geostationary satellite of mass 250 kg orbits the Earth above the Equator.
- State the period of the orbit.
 - Calculate the height of the satellite above the Equator.
 - Calculate the linear speed of the satellite.
 - Calculate the centripetal force acting on the geostationary satellite.
11. The Moon is a satellite of the Earth.
Calculate the period of the Moon's orbit in days.
12. Show by calculation that the Earth takes approximately 365 days to orbit the Sun.
13. The table shows some information about four of Saturn's moons.

Moon name	Titan	Rhea	Dione	Enceladus
Mean orbit radius R (km)	1.22×10^6	5.27×10^5	3.77×10^5	2.38×10^5
Orbit period T (days)	16	4.5	2.7	1.37

Show that T^2 is directly proportional to R^3 .

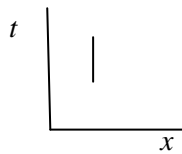
14. Calculate the gravitational potential at a point:
- on the Earth's surface
 - 800 km above the Earth's surface
 - 100 km above the Moon's surface.
15. Calculate the gravitational potential energy of:
- a 250 kg rocket on the Earth's surface
 - a 500 kg satellite orbiting 350 km above the Earth's surface
 - a 75 kg astronaut on the surface of the Moon.
16. A satellite of mass 450 kg has a gravitational potential energy of -2.3×10^{10} J.
Calculate the height of the satellite above the Earth's surface.
17. A geostationary satellite of mass 2000 kg orbits 3.6×10^7 m above the Earth's surface.
Calculate:
- the gravitational potential energy of the satellite
 - the kinetic energy of the satellite
 - the total energy of the satellite.
18. A satellite of mass 270 kg orbits the Earth at a height of 320 km above the Earth's surface.

Calculate the energy required to change the satellite's orbit to a height of 460 km above the Earth's surface.

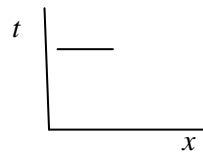
19. A satellite of mass m orbits a planet of mass M with a radius of R .
Show that the total energy of the satellite in orbit is $-\frac{1}{2} \frac{GMm}{R}$
20. Derive an expression for the escape velocity from a planet in terms of the mass m and radius r of the planet.
21. Calculate the escape velocity from the surface of:
- (a) the Earth
 - (b) the Moon
 - (c) Mars.
22. Calculate the escape velocity for a satellite that is in orbit 550 km above the Earth's surface.
23. The escape velocity from the surface of the Moon is $2.4 \times 10^3 \text{ m s}^{-1}$.
- (a) An object is projected from the surface of the Moon with a speed of $2.0 \times 10^3 \text{ m s}^{-1}$. Calculate the maximum height reached above the Moon's surface.
 - (b) An object is projected from the surface of the Moon with a speed of $2.8 \times 10^3 \text{ m s}^{-1}$. Calculate the speed of the object after it leaves the Moon's gravitational field.

Space and time

1. What is meant by an inertial frame of reference?
2. What is meant by a non-inertial frame of reference?
3. Which frame of reference applies to the theory of special relativity (studied in Higher Physics)?
4. At what range of speeds do the results obtained by the theory of special relativity agree with those of Newtonian mechanics?
5. How does the equivalence principle link the effects of gravity with acceleration?
6. In which part of an accelerating spacecraft does time pass more slowly?
7. Does time pass more quickly or more slowly at high altitude in a gravitational field?
8. How many dimensions are normally associated with space-time?
9. Two space-time diagrams are shown, with a worldline on each. Write down what each of the worldlines describes.

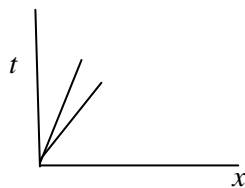


(a)

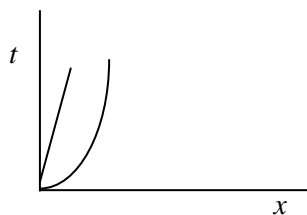


(b)

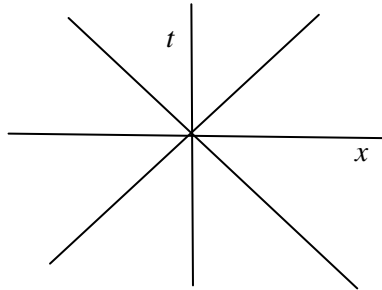
10. The space-time diagram shows two worldlines. Which worldline describes a faster speed? (These speeds are much less than the speed of light.)



11. Explain the difference between these two worldlines on the space-time diagram.



12. Explain what is meant by the term *geodesic*.
13. Copy the following space-time diagram.



Insert the following labels onto your diagram:

- the present
 - the future
 - the past
 - $v = c$
 - $v < c$
 - $v > c$.
14. What effect does mass have on spacetime?
 15. Describe two situations where a human body experiences the sensation of force.
 16. How does general relativity interpret the cause of gravity?
 17. Mercury's orbit around the Sun could not be predicted accurately using classical mechanics. General relativity was able to predict Mercury's orbit accurately. Investigate this using a suitable search engine and write a short paragraph summarising your results.
 18. A star of mass 4.5×10^{31} kg collapses to form a black hole. Calculate the Schwarzschild radius of this black hole.
 19. A star of mass equivalent to six solar masses collapses to form a black hole. Calculate the Schwarzschild radius of this black hole.
 20. If our Sun collapsed to form a black hole, what would be the Schwarzschild radius of this black hole?
 21. If our Earth collapsed to form a black hole, what would be the Schwarzschild radius of this black hole?
 22. A star is approximately the same size as our Sun and has an average density of 2.2×10^3 kg m^{-3} . If this star collapsed to form a black hole, calculate the Schwarzschild radius of the black hole.

Stellar physics

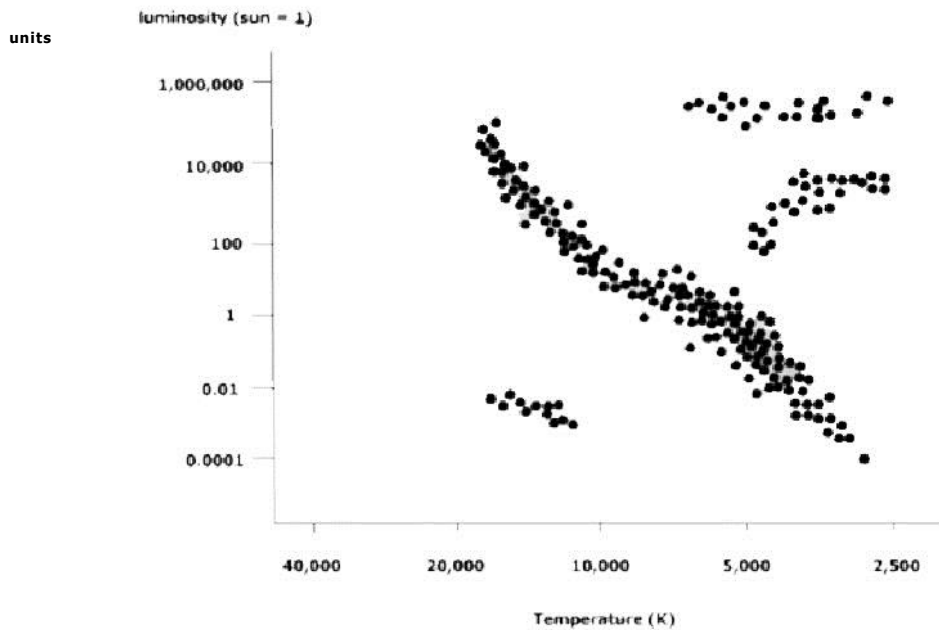
- A star emits electromagnetic radiation with a peak wavelength of 6.8×10^{-7} m.
 - Use Wien's law ($\lambda_{\max}T = 3 \times 10^{-3}$) to calculate the surface temperature of the star.
 - Calculate the power of the radiation emitted by each square metre of the star's surface where the star is assumed to be a black body.
Stefan–Boltzmann constant = $5.67 \times 10^{-8} \text{ J s}^{-1} \text{ m}^{-2} \text{ K}^{-4}$.
- The Sun has a radius of 7.0×10^8 m and a surface temperature of 5800 K.
 - Calculate the power emitted per m^2 from the Sun's surface.
 - Calculate the luminosity of the Sun.
 - Calculate the apparent brightness of the Sun as seen from the Earth.
- Three measurements of a distant star are possible from Earth. These measurements are:

apparent brightness	= $4.3 \times 10^{-9} \text{ W m}^{-2}$
peak emitted wavelength	= $2.4 \times 10^{-7} \text{ m}$
distance to star (parallax method)	= $8.5 \times 10^{17} \text{ m}$

 - Use Wien's law ($\lambda_{\max}T = 3 \times 10^{-3}$) to calculate the surface temperature of the star.
 - Calculate the energy emitted by each square metre of the star's surface per second.
 - Calculate the luminosity of the star.
 - Calculate the radius of the star.
- A star is 86 ly from Earth and has a luminosity of $4.8 \times 10^{28} \text{ W m}^{-2}$. Calculate the apparent brightness of the star.
- The apparent brightness of a star is $6.2 \times 10^{-8} \text{ W m}^{-2}$. The star is 16 ly from Earth. Calculate the luminosity of the star.
- A star with luminosity $2.1 \times 10^{30} \text{ W m}^{-2}$ has an apparent brightness of $7.9 \times 10^{-8} \text{ W m}^{-2}$ when viewed from Earth. Calculate the distance of the star from Earth:
 - in metres
 - in light years.
- A star with radius 7.8×10^8 m and surface temperature 6300 K has an apparent brightness of $1.8 \times 10^{-8} \text{ W m}^{-2}$. Calculate its distance from the Earth.
- A star with radius 9.5×10^9 m and surface temperature 5900 K is 36 ly from Earth. Calculate the apparent brightness of the star.
- Show mathematically that the luminosity of a star varies directly with the square of its radius and the fourth power of its surface temperature.

10. Show mathematically that the apparent brightness of a star varies directly with the square of its radius and the fourth power of its surface temperature and varies inversely with the square of its distance from the Earth.
11. Two stars, A and B, are the same distance from the Earth.
The apparent brightness of star A is $8.0 \times 10^{-12} \text{ W m}^{-2}$ and the apparent brightness of star B is $4.0 \times 10^{-13} \text{ W m}^{-2}$.
Show that star A has 20 times the luminosity of star B.
12. A star has half of our Sun's surface temperature and 400 times our Sun's luminosity.
How many times bigger is the radius of this star compared to the Sun?
13. Information about two stars A and B is given below.
- | | |
|-----------------------------|--|
| surface temperature of star | $A = 3 \times$ surface temperature of star B |
| radius of star | $A = 2 \times$ radius of star B |
- (a) How many times is the luminosity of star A greater than the luminosity of star B?
- (b) Stars A and B have the same apparent brightness from Earth.
Which star is furthest from Earth and by how many times?

14. The diagram shows one way of classifying stars. Each dot on the diagram represents a star.



- (a) What name is usually given to this type of diagram?
- (b) The stars are arranged into four main regions. Identify the region called:
 - (i) the main sequence
 - (ii) giants
 - (iii) super giants
 - (iv) white dwarfs.
- (c)
 - (i) In which of the regions on the diagram is the Sun?
 - (ii) The surface temperature of the Sun is approximately 5800 K. Explain why the scale on the temperature axis makes it difficult to identify which dot represents the Sun.
- (d) In which region would you find the following:
 - (i) a hot bright star
 - (ii) a hot dim star
 - (iii) a cool bright star
 - (iv) a cool dim star?
- (e) A star is cooler than, but brighter than the Sun.
 - (i) What can be deduced about the size of this star compared to the size of the Sun?
 - (ii) What region would this star be in?
- (f) A star is hotter than, but dimmer than, the Sun.
 - (i) What can be deduced about the size of this star compared to the size of the Sun?
 - (ii) What region would this star be in?
- (g) The Sun's nuclear fuel will be used up with time. What will then happen to the Sun's position in the above diagram?

Waves and Quanta

Quantum theory

1. The uncertainty in an electron's position relative to an axis is given as $\pm 5.0 \times 10^{-12}$ m.
Calculate the least uncertainty in the simultaneous measurement of the electron's momentum relative to the same axis.
2. An electron moves along the x -axis with a speed of 2.05×10^6 m s⁻¹ $\pm 0.50\%$.
Calculate the minimum uncertainty with which you can simultaneously measure the position of the electron along the x -axis.
3. An electron spends approximately 1.0 ns in an excited state.
Calculate the uncertainty in the energy of the electron in this excited state.
4. The position of an electron can be predicted to within ± 40 atomic diameters. The diameter of an atom can be taken as 1.0×10^{-10} m.
Calculate the simultaneous uncertainty in the electron's momentum.
5. Calculate the de Broglie wavelength of:
 - (a) an electron travelling at 4.0×10^6 m s⁻¹
 - (b) a proton travelling at 6.5×10^6 m s⁻¹
 - (c) a car of mass 1000 kg travelling at 120 km per hour.
6. An electron and a proton both move with the same velocity of 3.0×10^6 m s⁻¹.
Which has the larger de Broglie wavelength and by how many times larger (to 2 significant figures)?
7. Gamma rays have an energy of 4.2×10^{-13} J.
 - (a) Calculate the wavelength of the gamma rays.
 - (b) Calculate the momentum of the gamma rays.
8. An electron is accelerated from rest through a p.d. of 200 V in a vacuum.
 - (a) Calculate the final speed of the electron.
 - (b) Calculate the de Broglie wavelength of the electron at this speed.
 - (c) Would this electron show particle or wave-like behaviour when passing through an aperture of diameter 1 mm?
9. An electron is accelerated from rest through a p.d. of 2.5 kV.
Calculate the final de Broglie wavelength of this electron.
10. An electron microscope accelerates electrons until they have a wavelength of 40 pm (40×10^{-12} m).
Calculate the p.d. in the microscope required to do this assuming the electrons start from rest.

11. Relativistic effects on moving objects can be ignored provided the velocity is less than 10% of the speed of light.

What is the minimum wavelength of an electron produced by an electron microscope where relativistic effects can be ignored?

12. An electron moves round the nucleus of a hydrogen atom.

(a) Calculate the angular momentum of this electron:

- (i) in the first stable orbit
- (ii) in the third stable orbit.

(b) Starting with the relationship

$$mrv = \frac{nh}{2\pi}$$

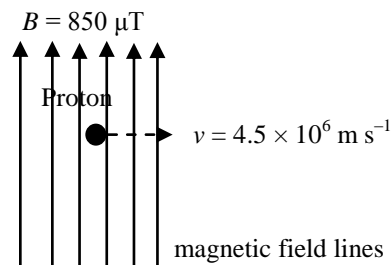
show that the circumference of the third stable orbit is equal to three electron wavelengths.

(c) The speed of an electron in the second stable orbit is $1.1 \times 10^6 \text{ m s}^{-1}$.

- (i) Calculate the wavelength of the electron.
- (ii) Calculate the circumference of the second stable orbit.

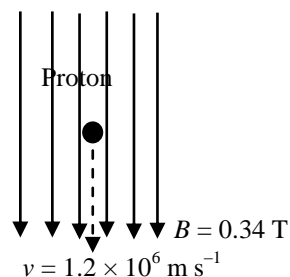
Particles from space

- An electron moves with a speed of $4.8 \times 10^6 \text{ m s}^{-1}$ at right angles to a uniform magnetic field of magnetic induction 650 mT.
Calculate the magnitude of the force acting on the electron.
- A proton moves with a speed of $3.0 \times 10^4 \text{ m s}^{-1}$ at right angles to a uniform magnetic field. The magnetic induction is 0.8 T. The charge on the proton is $+1e$.
Calculate the magnitude of the force acting on the proton.
- A neutron moves at right angles to a uniform magnetic field.
Explain why the neutron's motion is unaffected by the magnetic field.
- (a) A proton moves through a uniform magnetic field as shown in the diagram.



Calculate the magnetic force exerted on the proton.

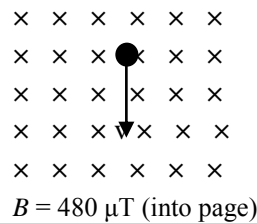
- (b) Another proton moves through this uniform magnetic field.



What is the magnetic force exerted on the proton? Explain your answer.

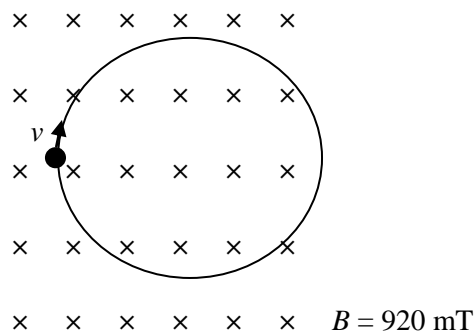
- An electron experiences a force of $2.5 \times 10^{-13} \text{ N}$ as it moves at right angles to a uniform magnetic field of magnetic induction 350 mT.
Calculate the speed of the electron.
- A muon experiences a force of $1.5 \times 10^{-16} \text{ N}$ when travelling at a speed of $2.0 \times 10^7 \text{ m s}^{-1}$ at right angles to a magnetic field. The magnetic induction of this field is $4.7 \times 10^{-5} \text{ T}$.
What is the magnitude of the charge on the muon?

7. An alpha particle is a helium nucleus containing two protons and two neutrons. The alpha particle experiences a force of 1.4×10^{-12} N when moving at 4.8×10^5 m s⁻¹ at right angles to a uniform magnetic field.
Calculate the magnitude of the magnetic induction of this field.
8. An electron moves at right angles to a uniform magnetic field of magnetic induction 0.16 T. The speed of the electron is 8.2×10^6 m s⁻¹.
- Calculate the force exerted on the electron.
 - Explain why the electron moves in a circle.
 - Calculate the radius of this circle.
9. A proton moves through the same magnetic field as in question 8 with the same speed as the electron (8.2×10^6 m s⁻¹).
Calculate the radius of the circular orbit of the proton.
10. An electron moves with a speed of 3.8×10^6 m s⁻¹ perpendicular to a uniform magnetic field.



Calculate:

- the radius of the circular orbit taken by the electron
 - the central force acting on the electron.
11. An alpha particle travels in a circular orbit of radius 0.45 m while moving through a magnetic field of magnetic induction 1.2 T. The mass of the alpha particle is 6.645×10^{-27} kg. Calculate:
- the speed of the alpha particle in the orbit
 - the orbital period of the alpha particle
 - the kinetic energy of the alpha particle in this orbit.
12. A proton moves in a circular orbit of radius 22 mm in a uniform magnetic field as shown in the diagram.



Calculate the speed of the proton.

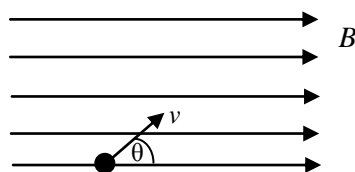
13. An electron moves with a speed of $5.9 \times 10^5 \text{ m s}^{-1}$ in a circular orbit of radius $5.5 \text{ }\mu\text{m}$ in a uniform magnetic field.

Calculate the magnetic induction of the magnetic field.

14. A sub-atomic particle moves with a speed of $2.09 \times 10^6 \text{ m s}^{-1}$ in a circular orbit of radius 27 mm in a uniform magnetic field. The magnetic induction is 0.81 T .

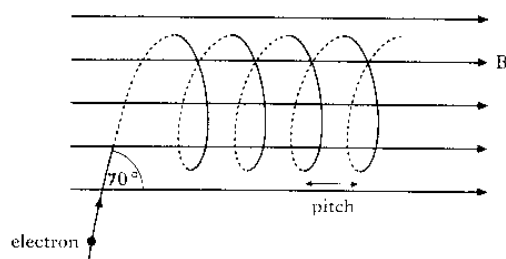
Calculate the charge to mass ratio of the sub-atomic particle and suggest a name for the particle. Give a reason for your answer.

15. A charged particle enters a uniform magnetic field with a velocity v at an angle θ as shown.



- (a) Write down an expression for the horizontal component of velocity.
 (b) Write down an expression for the vertical component of velocity.
 (c) Which of these components will stay unchanged as the charged particle continues its journey? Give a reason for your answer.

16. An electron travelling at a constant speed of $6.8 \times 10^6 \text{ m s}^{-1}$ enters a uniform magnetic field at an angle of 70° as shown and subsequently follows a helical path.



The magnetic induction is 230 mT .

Calculate:

- (a) the component of the electron's initial velocity parallel to B;
 (b) the component of the electron's initial velocity perpendicular to B;
 (c) the central force acting on the electron;
 (d) the radius of the helix;
 (e) the period of electron rotation in the helix;
 (f) the pitch of the helix.

17. A proton travelling at $5.8 \times 10^5 \text{ m s}^{-1}$ enters a uniform magnetic field at an angle of 40° to the horizontal (similar to the diagram in question 16). The proton subsequently follows a helical path.

The magnetic induction is 0.47 T.

Calculate:

- (a) the component of the proton's initial velocity parallel to B;
- (b) the component of the proton's initial velocity perpendicular to B;
- (c) the central force acting on the proton;
- (d) the radius of the helix;
- (e) the period of proton rotation in the helix;
- (f) the pitch of the helix.

18. An electron travelling at $1.3 \times 10^7 \text{ m s}^{-1}$ enters a uniform magnetic field at an angle of 55° and follows a helical path similar to that shown in question 16. The magnetic induction is 490 mT.

Calculate:

- (a) the radius of the helix
- (b) the pitch of the helix.

19. Explain why most charged particles from the Sun enter the Earth's atmosphere near the north and south poles.
20. Explain what causes the Aurora Borealis to occur.

Simple Harmonic Motion 1

1. (a) An object undergoes simple harmonic motion. State the condition which must apply to the unbalanced force acting on the object.
(b) Give three examples of simple harmonic motion (SHM).
2. (a) State the equation which defines SHM.
(b) (i) Show by differentiation that *each* of the following is a solution of the equation for SHM:
$$y = a \cos \omega t \quad \text{and} \quad y = a \sin \omega t.$$

(ii) State the condition under which the equation for SHM is given by *each* of the following:
$$y = a \cos \omega t \quad \text{and} \quad y = a \sin \omega t$$

(c) Derive the equation for the velocity $v = \pm \sqrt{a^2 - y^2}$ using:
(i) $y = a \cos \omega t$
(ii) $y = a \sin \omega t$.
3. An object moves with SHM with a frequency of 5 Hz and an amplitude of 40 mm.
(a) Find the acceleration at the centre and extremities of the motion.
(b) Determine the velocity at the centre and extremities of the motion.
(c) Calculate the acceleration and velocity at a point midway between the centre and extremity of the motion.
4. A horizontal platform oscillates vertically with SHM with a slowly increasing amplitude. The period of the oscillations is 0.10 s.
What is the maximum amplitude which will allow a mass resting on the platform to remain in contact with the platform?
5. (a) Derive expressions for the kinetic energy and potential energy of a particle executing SHM.
(b) An object of mass 0.20 kg oscillates with SHM with an amplitude of 100 mm. The frequency of the oscillations is 0.50 Hz.
(i) Calculate the maximum value of the kinetic energy of the object. State where this occurs.
(ii) State the minimum value of the kinetic energy. State where this occurs.
(iii) Find the maximum value of the potential energy of the object. State where this occurs.
(iv) Calculate the potential energy and the kinetic energy at a point mid way between the centre and extremity of the motion.
(v) What can you state about the value of the sum of the potential energy and the kinetic energy at any point?
6. The displacement, y , in mm of a particle is given by $y = 0.44 \sin 28t$.
(a) Find the amplitude of the motion.
(b) Find the frequency of the motion.
(c) Find the period of the motion.
(d) Find the time taken for the particle to move a distance of 0.20 mm from the equilibrium position.

7.
 - (a) What effect does damping have on an oscillatory system?
 - (b) Briefly explain the terms critical damping and overdamping.
 - (c) Give two examples where damping is useful.

Simple Harmonic Motion 2

- The displacement, in cm, of a particle is given by the equation: $y = 4 \cos 4\pi t$.
 - State the amplitude of the motion.
 - Calculate the frequency, and hence the period, of the oscillation.
 - Calculate the location of the particle, in relation to its rest position, when;
 - $t = 0$
 - $t = 1.5$ s.
- A body, which is moving with SHM, has an amplitude of 0.05 m and a frequency of 40 Hz.
 - Find the period of the motion.
 - State an appropriate equation describing the motion.
 - Calculate the acceleration at the mid-point of the motion **and** at the position of maximum amplitude.
 - Calculate the maximum speed of the body and state at which point in the motion this speed occurs.
- An object of mass 0.50 kg moves with SHM. The amplitude and period of the motion are 0.12 m and 1.5 s respectively. Assume that the motion starts with $a = + 0.12$ m.

From this information, calculate:

- the position of the object when $t = 0.40$ s
 - the force (magnitude and direction) acting on this object when $t = 0.40$ s
 - the minimum time needed for the object to travel from its starting point to a point where the displacement is $- 0.06$ m.
- A prong of a tuning fork, which can be assumed to be moving with simple harmonic motion, has the following equation governing its motion:
$$y = 2.0 \sin (3.22 \times 10^3 t)$$
 where y is in mm.
 - Find the maximum amplitude and the frequency of the tuning fork's motion.
 - Calculate the maximum acceleration of the prong on the tuning fork.
 - On graph paper, draw the variation of displacement against time for the first two cycles of the motion. Assume that the motion starts from the equilibrium position.
 - As the sound of a tuning fork dies away, the frequency of the note produced does not change. What conclusion can we draw about the period of this, and indeed any object, moving with SHM?
 - A sheet of metal is clamped in the horizontal plane and made to vibrate with SHM in the vertical plane with a frequency of 40 Hz. When some sand grains are sprinkled on to the plate, it is noted that the sand grains can lose contact with the sheet of metal. This occurs when the acceleration of the SHM is $\geq 10 \text{ m s}^{-2}$. Calculate the maximum amplitude of the motion for which the sand will always be in contact with the metal sheet.

- 6 A vertical spring stretches 0.10 m when 1.2 kg mass is allowed to hang from the end of the spring.
- Calculate the spring constant, k , given by these figures.
 - The mass is now pulled down a distance of 0.08 m below the equilibrium position and released from rest.
 - State the amplitude of the motion.
 - Calculate the period **and** the frequency of the motion.
 - Find the maximum speed of the mass **and** the total energy of the oscillating system.
- 7 A block of mass 5.0 kg is suspended from a spring which has a force constant of 450 N m^{-1} . A dart which has a mass of 0.060 kg is fired into the block from below with a speed of 120 m s^{-1} , along the vertical axis of the spring. The dart embeds in the block.
- Find the amplitude of the resulting simple harmonic motion of the spring/block system.
 - What percentage of the original kinetic energy of the dart appears as energy in the oscillating system?
8. Explain what is meant by the terms 'damping' and 'critical damping' when applied to oscillating systems.

Waves 1

1. (a) State the relationship between the intensity and the amplitude of a wave.
(b) The amplitude of a wave increases ninefold.
What is the change in the intensity?
2. 'All waveforms can be described by the superposition of sine or cosine waves'.
Explain what is meant by this statement using either a square wave or a sawtooth wave as an example.
3. (a) The relationship $y = a \sin 2\pi(ft - x/\lambda)$ represents a travelling wave.
State clearly the meaning of each symbol in this equation.
(b) A travelling wave is represented by the relationship $y = 0.60 \sin \pi(150t - 0.40x)$ where standard SI units are used throughout.
 - (i) What is the amplitude of the wave?
 - (ii) Determine the frequency of the wave.
 - (iii) State the period of the wave.
 - (iv) Calculate the wavelength of the wave.
 - (v) What is the wave speed?
4. Two waves are represented by the relationships:
 $y_1 = 4.0 \sin 2\pi(8t - 5x)$ and $y_2 = 4.0 \sin \pi(16t - 21x)$ respectively.
 - (a) Which of the following quantities are the same for the two waves:
amplitude, frequency, wavelength, period.
 - (b) Are the two waves in phase? You must justify your answer.
5. (a) Explain what is meant by a 'stationary wave'.
(b) Define the terms 'nodes' and 'antinodes'.

Waves 2

- 1 A travelling wave is represented by the equation $y = 3 \sin 2\pi(10t - 0.2x)$ where y is in cm. Calculate, for this wave:
- the amplitude;
 - the frequency;
 - the wavelength;
 - the speed.

- 2 Write the equation for a plane sinusoidal wave travelling in the + x direction which has the following characteristics:
amplitude = 0.30 m, wavelength = 0.50 m and frequency = 20 Hz.

- 3 A travelling wave is represented by the following equation:

$$y_1 = 0.20 \sin (220\pi t - 30\pi x) \quad (i)$$

where y_1 and x are measured in m from the origin.

Write the equation for the displacement, y_2 , of a wave travelling in the opposite direction which has twice the frequency and double the amplitude of the wave represented by equation (i) above.

- 4 The equation of a transverse wave on a stretched string is represented by:

$$y = 0.04 \sin \left[2\pi \left(\frac{t}{0.04} - \frac{x}{2.0} \right) \right] \quad \text{where } y \text{ and } x \text{ in metres and } t \text{ in seconds.}$$

- What is the amplitude of the wave?
 - Calculate the wavelength of the wave.
 - What is the frequency of the wave?
 - Describe the movement of any particle of the string over one complete period, T , of the wave.
- 5 The equation of a transverse wave travelling in a rope is given by:
 $y = 0.01 \sin \pi (2.0 t - 0.01 x)$ where y and x in metres and t in seconds.
- Calculate the velocity of the wave in the x -direction.
 - Find the maximum **transverse** speed of a particle in the rope.

- 6 The following equation represents a wave travelling in the positive x -direction

$$y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$$

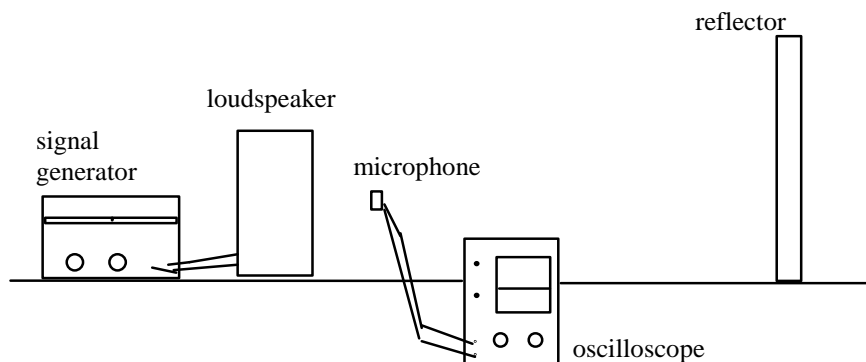
Using the relationships $f = 1/T$, $v = f\lambda$, and $k = 2\pi/\lambda$, show that the following are also possible equations for this wave.

- (a) $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$ (b) $y = a \sin (\omega t - kx)$
- (c) $y = a \sin 2\pi f \left(t - \frac{x}{v} \right)$ (d) $y = a \sin \frac{2\pi}{\lambda} (vt - x)$

- 7 A wave of frequency 500 Hz has a velocity of 350 m s^{-1} .

- How far apart are two points which are 60° i.e. $\frac{\pi}{3}$ out of phase?

- (b) What is the phase difference between two displacements at the same point, at a time separation of 0.001 s?
- 8 A progressive wave and a stationary wave each have the same frequency of 250 Hz and the same velocity of 30 m s^{-1} .
- (a) Calculate the phase difference between two vibrating points on the progressive wave which are 10 cm apart.
- (b) State the equation for the travelling wave if its amplitude is 0.03 m.
- (c) Calculate the distance between the nodes of the stationary wave.
- 9 (a) Explain what is meant by a 'travelling wave' and a 'stationary wave'. State clearly the differences between the two.
- (b) Describe a method involving the formation of standing waves which you could use to measure the wavelength of microwaves. In your answer you should include:
- a sketch of any apparatus you would use;
 - details of measurements taken;
 - details of how you would arrive at a final answer.
- 10 (a) The sketch below shows an experimental arrangement to measure the wavelength of sound waves coming from a loudspeaker.



The oscilloscope trace shows the level of sound picked up by the microphone which is moved between the loudspeaker and the reflector.

In one particular trial it was noted that the microphone travelled a distance of 0.24 m between adjacent maxima. The signal generator was set at 700 Hz.

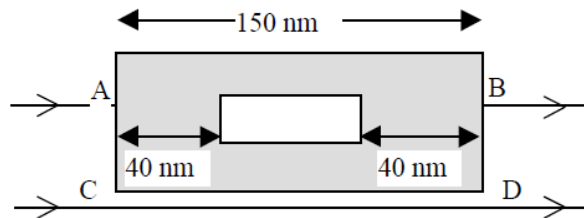
Calculate:

- (i) the wavelength and
- (ii) the velocity of the sound wave emitted from the loudspeaker.
- (b) Another loudspeaker is connected in parallel with the first and the two sound waves allowed to overlap. The two speakers are facing in the same direction and the reflector is removed.

Describe and explain what a listener would hear as he walks across in front of the two speakers.

Interference – division of amplitude 1

- State the condition for two light beams to be coherent.
 - Explain why two light beams, of the same frequency, but from different sources are unlikely to be coherent.
 - Can two loudspeakers connected to the same signal generator emit coherent beams of sound waves? Explain your answer.
- Define the term optical path difference.
 - State the relationship between the optical path difference and phase difference.
 - A hollow air filled perspex microfibre is shown below. Light of wavelength 700 nm passes through and around the microfibre.

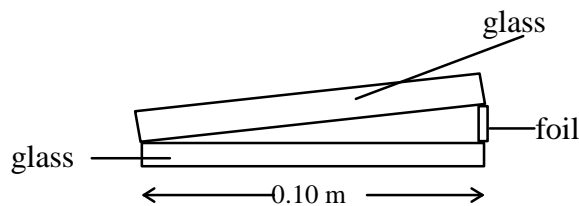


- Determine the optical path length between AB.
 - A ray of light follows the path AB above. Another ray follows the path CD, just outside the block.
What is the phase difference between the two rays?
- Light in air is reflected from a glass surface. What is the change in phase of the light waves?
 - What change in phase occurs when light in glass is reflected at a glass/water boundary back into the glass.
 - A thin parallel sided film is used to produce interference fringes.
 - Using the thin film as an example, explain the term 'interference produced by division of amplitude'. Include a sketch of the path of the light rays through the film
 - State the condition for a minimum to be produced in the fringes formed by reflection from the film of monochromatic light of wavelength λ .
 - What is the effect on the fringe pattern when the thickness of the film increases?
 - Derive the expression for the distance between the fringes which are formed by reflection of light from a thin wedge.
 - Two glass slides are 100 mm long. A wedge is formed with the slides by placing the slides in contact at one end. The other ends of the slide are separated by a piece of paper 30 μm thick. Interference fringes are observed using light of wavelength 650 nm. Calculate the separation of the fringes.
 - When looking at a slightly different part of the fringe pattern the fringes are observed to be slightly closer together. What does this imply about the paper.
You must justify your answer.

6. (a) Derive the expression $d = \lambda/4n$ for the thickness of a non-reflecting coating.
- (b) What thickness of coating is required to give non-reflection in green light of wavelength 540 nm for a lens of refractive index 1.53.
- (c) *Explain why some lenses with a non-reflective coating appear coloured.*

Interference – division of amplitude 2

- To observe interference effects with light waves the sources must be coherent.
 - Explain carefully what is meant by coherent waves.
 - Explain why the conditions for coherence are usually more difficult to satisfy for light than for sound or microwaves.
- Explain what is meant by division of amplitude.
 - Explain why an extended source can be used in experiments which involve division of amplitude.
- An air wedge 0.10 m long is formed by two glass plates in contact at one end and separated by a thin piece of foil at the other end as shown below.

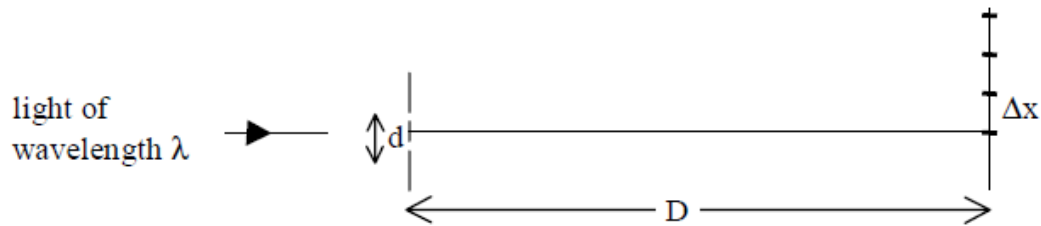


Interference fringes are observed in reflected light of wavelength 6.9×10^{-7} m. The average fringe separation is 1.2×10^{-3} m.

- Explain how the fringes are formed.
 - Calculate the thickness of the foil.
 - The foil is now heated and its thickness increases by 10%. Calculate the new separation of the fringes.
- Derive the expression for the thickness of a non-reflecting coating on a lens. Your answer should be in terms of the incident wavelength and the refractive index of the coating.
 - Calculate the thickness of the coating required to produce destructive interference at a wavelength of 4.80×10^{-7} m, given that the refractive index of the coating is 1.25.
 - A lens is coated with a thin transparent film to reduce reflection of red light of wavelength 6.7×10^{-7} m. The film has a refractive index of 1.30. Calculate the required thickness of the film.
 - A soap film of refractive index 1.3 is illuminated by light of wavelength 6.2×10^{-7} m. The light is incident normally on the soap film. Calculate the minimum thickness of soap film which gives no reflection.

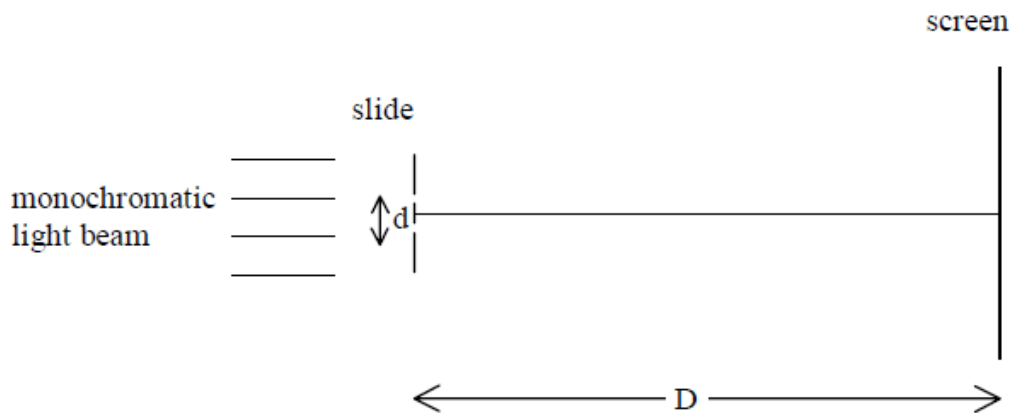
Interference – division of wavefront 1

- An interference pattern is obtained by division of wavefront. What is meant by 'division of wavefront'.
 - Why must the source be a point source to produce interference by division of wavefront?
 - Explain why an extended source can be used to produce an interference pattern by division of amplitude.
- The diagram below shows the set up for a Young's double slit experiment.



- Derive the expression $\Delta x = \frac{\lambda D}{d}$ for the fringe spacing.
 - State any assumptions made in the above derivation.
- Two parallel slits have a separation of (0.24 ± 0.01) mm. When illuminated by light an interference pattern is observed on a screen placed (3.8 ± 0.1) m from the double slits. The fringe separation is observed to be (9.5 ± 0.1) mm.
 - Calculate the wavelength of the light used.
 - Determine the uncertainty in this wavelength.

4. Two slits, of separation d , are made on a slide. The slide is illuminated by monochromatic light as shown below.

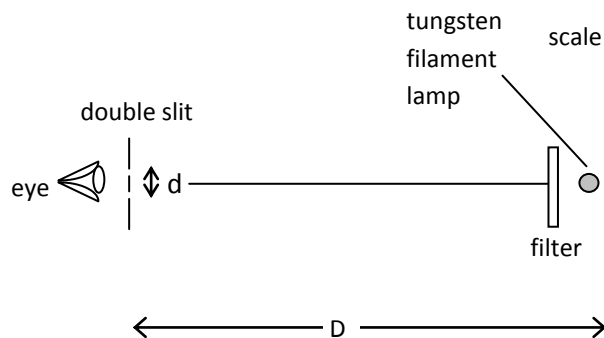


Fringes are observed on the screen.

- The fringe spacing is observed to be too small to make accurate measurements. State one way of increasing the fringe spacing using this apparatus.
- The light beam is replaced by one of light of a higher wavelength. What effect will this have on the fringe spacing?
- The slide is removed and replaced with another slide. The second slide has two slits with a smaller separation, d . What effect does this have on the fringe pattern?
- What can be used to measure the slit separation?
- Describe how the fringe separation could be measured.

Interference – division of wavefront 2

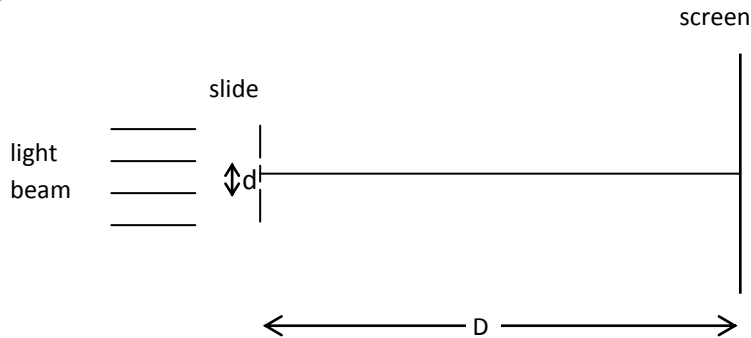
- There are two methods of producing interference with light, namely; division of amplitude and division of wavefront.
Give an example of each of the above and explain, with the aid of diagrams, the difference between the two methods.
- White light illuminates two narrow closely spaced slits. An interference pattern is seen on a distant screen.
 - Explain how the interference pattern occurs.
 - The white fringes have coloured edges. Explain how this occurs.
- A laser beam is directed towards a double slit and an interference pattern is produced on a screen which is 0.92 m from the double slit. The separation of the double slit is 2.0×10^{-4} m. The wavelength of the light used is 695 nm.
 - Calculate the separation of the bright fringes on the screen.
 - The double slit is now replaced with a different double slit of separation 1.0×10^{-4} m. State and explain what effect this change will have on the interference pattern.
- Two parallel slits have a separation of 5.0×10^{-4} m. When illuminated by light of unknown wavelength an interference pattern is observed on a screen placed 7.2 m from the double slit. The separation of the bright fringes on the screen is 8 mm.
Calculate the wavelength of the light used.
- A pupil holds a double slit in front of his eye and looks at a tungsten lamp with a scale immediately behind it.



- A red filter is placed in front of the lamp. Describe what he sees and explain in terms of waves how this arises.
- The red filter is then replaced by a blue one. Explain any difference in fringe separation with blue and with red.
- Explain why the fringes have coloured edges when no filter is used.

- (d) With the red filter in place, the student estimates the apparent separation of the bright fringes to be 5.0 mm when the distance D is 2.0 m. The slit separation is 0.25 mm. Calculate the wavelength of the light passing through the filter from these measurements.

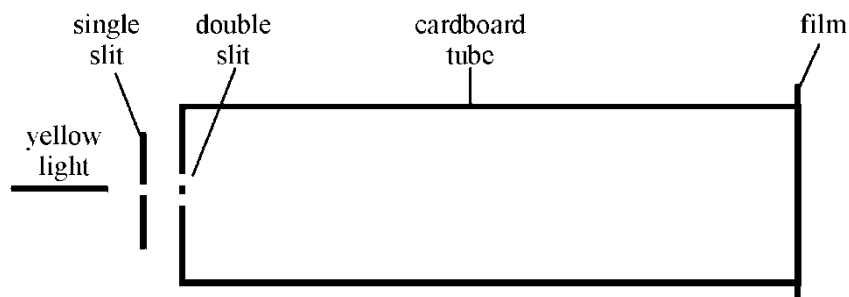
- 6 In a Young's slit experiment designed to demonstrate the interference of light, two parallel slits scratched on a blackened microscope slide are illuminated by an intense beam of monochromatic light.



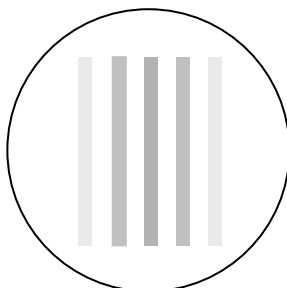
Bright fringes with an average separation Δx are observed on a distant screen.

- (a) State the effect of
- bringing the screen closer to the slits
 - reducing the separation of the slits.
- (b) Explain the effect on the Young's interference pattern of
- covering one of the slits
 - using light of a longer wavelength
 - using white light.
- (c) Two parallel slits 0.5 mm apart are found to produce fringes with an average separation of 10 mm on a screen placed at a distance of 8 m from the double slit. What do these figures give for the wavelength of the incident light?
- (d) In the practical determination of this wavelength three distances have to be measured. By considering each measurement in turn, explain which one would be the most critical in obtaining a reasonably accurate result.

- 7 A beam of yellow light from a single slit falls on a double slit, which is mounted on the end of a cardboard tube as shown below.



The interference pattern formed is recorded on a piece of photographic film placed over the end of the tube. When the film is developed a series of black lines can be seen. One such film is shown below.



- (a) In one experiment a student obtains the following results:
- | | |
|-----------------------------------|------------------------------|
| distance between dark lines | $= 7 \pm 1 \text{ mm}$ |
| separation of double slit | $= 0.20 \pm 0.01 \text{ mm}$ |
| distance from double slit to film | $= 2.40 \pm 0.01 \text{ m}$ |

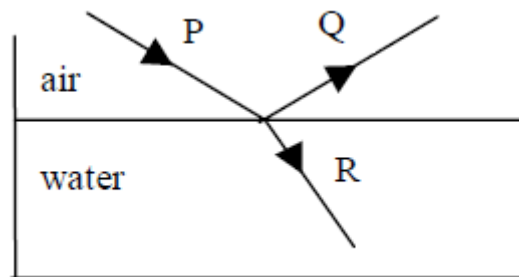
From these measurements, calculate:

- (i) the wavelength of yellow light;
(ii) the uncertainty in this value.
- (b) (i) Describe one method of measuring the double slit separation to the stated degree of accuracy.
(ii) Give one way in which the uncertainty in the measurement of the separation of the black lines on the film could be reduced.
- (c) In each case, state and explain the effect on the film pattern, when:
- (i) the double slits are closer together;
(ii) blue light is used instead of yellow light;
(iii) one of the slits is covered.

Polarisation 1

- Explain the difference between linearly polarised and unpolarised waves.
 - Describe how an unpolarised wave can be linearly polarised using a polaroid filter.
 - Describe how a 'polariser' and 'analyser' can prevent the transmission of light.
- Monochromatic light is incident at a boundary between air and another medium. The reflected light is found to be polarised.
 - What information does this provide about the nature of the medium?
 - Derive the expression relating the polarising angle and the refractive index of the medium for this light.
 - State the other common name for the polarising angle.
- Light is incident on a rectangular block of perspex
 - Draw a sketch to show the position of the polarising angle for perspex.
 - Mark on your sketch for part (a) the value of the polarising angle.
- Explain how sunglasses can remove glare.
- The refractive index of a liquid is 1.45.
 - Calculate the polarising angle for this liquid.
 - Determine the value of the angle of refraction for this polarising angle.
- The critical angle in a certain glass is 40.5° .
What is the polarising angle for this glass?
- A spectrum can be produced by a prism because the refractive index changes with the frequency of light.
What effect will an increase in the frequency of light have on the polarising angle?
You must justify your answer.

8. Light is incident on a water surface as shown below.



The angle between the ray Q and R is 90° .

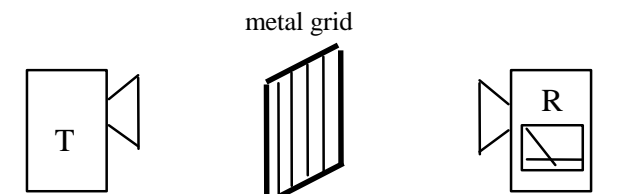
- (a) The ray Q is observed through a sheet of polaroid. The polaroid is rotated. Describe and explain what is observed.
- (b) Calculate the polarising angle for water.
- (c) Copy the diagram and label in the correct places the values of the angle of incidence and angle of refraction.

Polarisation 2

- Light is reflected from a smooth glass surface at an angle which produces plane polarised light. The refractive index of the glass is 1.52.
 - Calculate the angles of incidence and refraction.
 - Describe how you would prove that the reflected light was plane polarised.
- A student investigates the glare from a smooth water surface using a polaroid filter as an analyser. She finds that the angle of incidence required to produce plane polarised light is 52° .
 - State the angle of refraction.
 - Calculate the refractive index of water given by these figures.
- A beam of white light is reflected from the flat surface of a sample of crown glass. The information below gives the variation of refractive index with wavelength for crown glass.

refractive index	wavelength / nm
1.52	650 - red
1.53	510 - green
1.54	400 - violet

- Calculate the range of polarising angle for incident white light.
 - Calculate the maximum angle of refraction.
- A student sets up the following microwave apparatus.



The transmitter, T, sends out microwaves of wavelength 0.028 m.

As the metal grid is rotated through 360° , the reading on the receiver, R, becomes a maximum and then a minimum and then a maximum again.

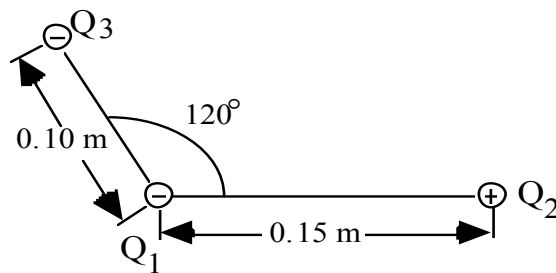
- Calculate the frequency of the microwaves.
 - Explain fully the behaviour of the reading on the receiver as the metal grid is rotated.
 - Another student sets up a small portable television in front of the window in his new flat. He finds that unless he raises the metal venetian blind at the window the reception on the television is very poor.
Explain why the reception is so poor in this situation.
- Monochromatic light is travelling into a medium and is reflected at the boundary with air. The critical angle for this light in the medium is 38° .
Calculate the polarising angle?

- 6 (a) What is meant by the polarising angle i_p ?
- (b) State another name for this angle i_p .
- (c) Derive the relationship between the polarising angle and the refractive index.
- (d) A beam of white light is incident on a flat glass surface at an angle of 56° . The reflected beam is plane polarised.
- (i) Calculate the angle of refraction in the glass
- (ii) Calculate the refractive index of the glass.
- 7 (a) Sunlight is reflected off the smooth water surface of an unoccupied swimming pool. The refractive index of water is 1.33.
- (i) At what angle of reflection is the sunlight completely plane polarised?
- (ii) What is the corresponding angle of refraction for the sunlight that is refracted into the water.
- (b) At night an underwater floodlight is turned on the pool.
- (i) At what angle of reflection is the floodlight completely plane polarised?
- (ii) What is the corresponding angle of refraction for the light that is refracted into the air?

Electromagnetism

Coulomb's Inverse Square Law

- 1 A charge of $+ 2.0 \times 10^{-8} \text{ C}$ is placed a distance of 2.0 mm from a charge of $- 4.0 \times 10^{-8} \text{ C}$.
 - (a) Calculate the electrostatic force between the charges.
 - (b) The distance between the same charges is adjusted until the force between the charges is $1.0 \times 10^{-4} \text{ N}$.
Calculate this new distance between the charges.
- 2 Compare the electrostatic and gravitational forces between a proton and electron which have an average separation of $2.0 \times 10^{-10} \text{ m}$.
- 3 Let us imagine the Earth (mass $6.0 \times 10^{24} \text{ kg}$) suddenly has an excess of positive charge and the Moon (mass $7.3 \times 10^{22} \text{ kg}$) suddenly has an equal excess of positive charge. Calculate the size of the charge required so that the electrostatic force balances the gravitational force.
- 4 The diagram below shows three charges fixed in the positions shown.



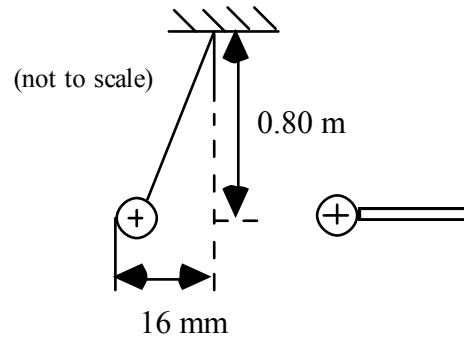
$$Q_1 = - 1.0 \times 10^{-6} \text{ C}, \quad Q_2 = + 3.0 \times 10^{-6} \text{ C} \quad \text{and} \quad Q_3 = - 2.0 \times 10^{-6} \text{ C}.$$

Calculate the resultant force on charge Q_1 . (Remember that this resultant force will have a **direction** as well as magnitude).

- 5 Two like charged spheres of mass 0.10 g, hung from the same point by silk threads are repelled from each other to a separation of 1.0 cm by the electrostatic force. The angle between one of the silk threads and the vertical is 5.7° .
 - (a) By drawing a force diagram, find the electrostatic force F_E between the spheres.
 - (b) Calculate the size of the charge on each sphere.
 - (c) The average leakage current from a charged sphere is $1.0 \times 10^{-11} \text{ A}$. How long would it take for the spheres to discharge completely?
 - (d) Describe how the two spheres may be given identical charges.

- 6 In an experiment to show Coulomb's Law, an insulated, light, charged sphere is brought close to another similarly charged sphere which is suspended at the end of a thread of length 0.80 m. The mass of the suspended sphere is 0.50 g.

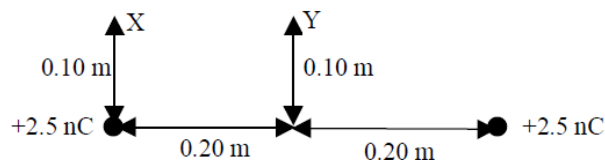
It is found that the suspended sphere is displaced to the left by a distance of 16 mm as shown below.



- (a) Make a sketch showing all of the forces acting on the suspended sphere.
- (b) Calculate the electrostatic force acting on the suspended sphere.
7. Explain how a charged strip of plastic can pick up small pieces of paper even although the paper has no net charge.

Electric fields and electrostatic potential

- A metallic sphere has a radius of 0.040 m. The charge on the sphere is $+30 \mu\text{C}$.
Determine the electric field strength
 - inside the sphere;
 - at the surface of the sphere;
 - at a distance of 1.0 m from the centre of the sphere.
- Describe, with the aid of a diagram, the process of charging by induction.
- What is meant by the 'electrostatic potential at a point'?
- State the expression for the electrostatic potential at a distance r from a point charge Q .
- Determine the electrostatic potential at a distance of 3.0 m from a point charge of $+4.0 \text{ nC}$.
- Point A is 2.0 m from a point charge of -6.0 nC . Point B is 5.0 m from the same point charge. Determine the potential difference between point A and point B.
- What is meant by an equipotential surface?
- A very small sphere carries a positive charge. Draw a sketch showing lines of electric field for this charge. Add lines of equipotential to your sketch using broken dashed lines.
- Two charges of $+4.0 \text{ nC}$ and -2.0 nC are situated 0.12 m apart.
Find the position of the point of zero electrostatic potential.
- Which of the following are vector quantities:
electrostatic force, electric field strength, electrostatic potential, permittivity of free space, electric charge, potential difference.
- Two charges each of $+2.5 \text{ nC}$ are situated 0.40 m apart as shown below.

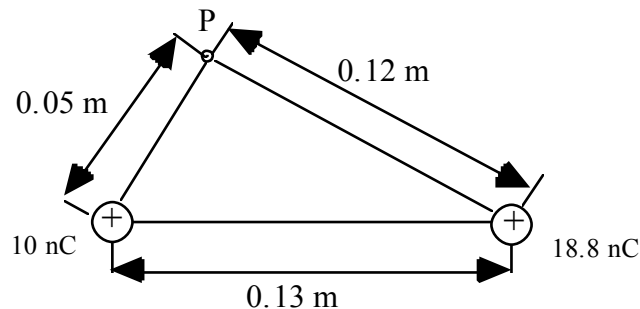


- What is the electrostatic potential at point X?
 - What is the electrostatic potential at point Y?
- Determine the potential difference between X and Y.

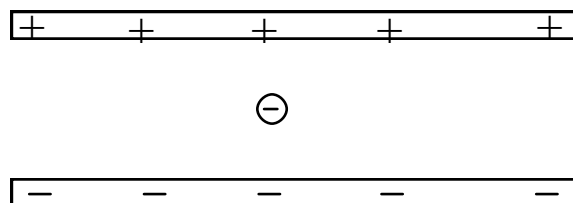
12. In a uniform electric field an electron gains 10^{-14} J when travelling between two points. What is the p.d. between these two points?

Electric Field Strength

- What is the electric field strength at a point where a small object carrying a charge of $4.0 \mu\text{C}$ experiences a force of 0.02 N ?
- Calculate the size of a point charge required to give an electric field strength of 1.0 N C^{-1} at a distance of 1.0 m from the point charge.
 - State the magnitude of the electric field strength at a distance of 2.0 m from the point charge.
- Calculate the electric field strength due to an α -particle at a point 5.0 mm from the α -particle.
 - How would the electric field strength calculated in (a) compare with the electric field strength at a point 5.0 mm from a proton?
- The diagram below shows two charges of $+10.0 \text{ nC}$ and $+18.8 \text{ nC}$ respectively separated by 0.13 m .



- Calculate the magnitude of the resultant electric field strength at the point P as shown in the diagram above.
 - Make a sketch like the one above and show the direction of the resultant electric field strength at the point P.
Angles are required on your sketch.
- A negatively charged sphere, of mass $2.0 \times 10^{-5} \text{ kg}$, is held stationary in the space between two charged metal plates as shown in the diagram below.

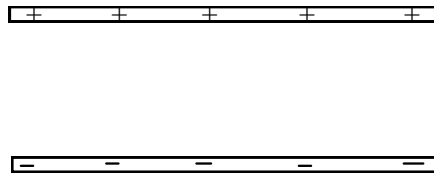


- The sphere carries a charge of $-5.0 \times 10^{-9} \text{ C}$. Calculate the size of the electric field strength in the region between the metal plates.
- Make a sketch of the two plates and the stationary charged sphere. Show the shape and direction of the resultant electric field in the region between the plates.

- 6 Two charges of $+8.0 \times 10^{-9} \text{ C}$ and $+4.0 \times 10^{-9} \text{ C}$ are held a distance of 0.20 m apart.
- Calculate the magnitude and direction of the electric field strength at the mid-point between the charges.
 - Calculate the distance from the $8.0 \times 10^{-9} \text{ C}$ charge at which the electric field strength is zero.
 - The $4.0 \times 10^{-9} \text{ C}$ charge has a mass of $5.0 \times 10^{-4} \text{ kg}$.
 - Calculate the magnitude of the electrostatic force acting on the charge.
 - Calculate the magnitude of the gravitational force acting on the mass.

7 Copy and complete the electric field patterns for:

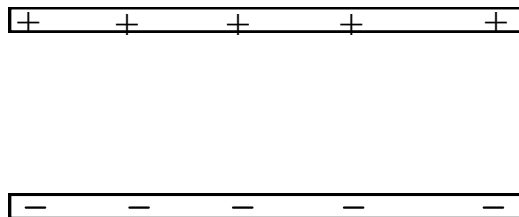
- (a) the electric field between two parallel plates which have equal but opposite charges



- (b) the electric field around 2 **unequal** but opposite charges.

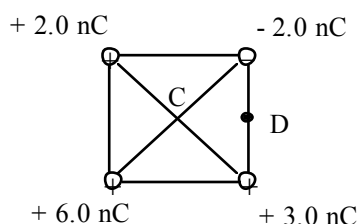


8 Draw electric field lines and equipotential surfaces for the two oppositely charged parallel plates shown in the sketch below. (Include the fringing effect usually observed near the edge of the plates).



Electrostatic Potential

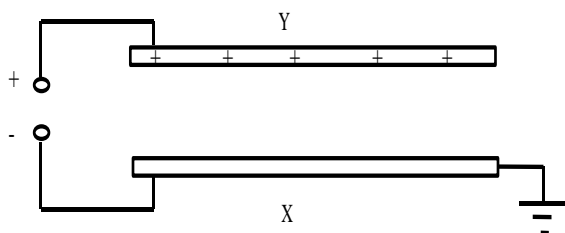
- Calculate the electrostatic potential at a point, P, which is at a distance of 0.05 m from a point charge of $+3.0 \times 10^{-9}$ C.
- Small spherical charges of +2.0 nC, -2.0 nC, +3.0 nC and +6.0 nC are placed in order at the corners of a square of diagonal 0.20 m as shown in the diagram below.



- Calculate the electrostatic potential at the centre, C, of the square.
 - D is at the midpoint of the side as shown.
Calculate the electrostatic potential **difference** between point C and point D.
- A hydrogen atom may be considered as a charge of $+1.6 \times 10^{-19}$ C separated from a charge of -1.6×10^{-19} C by a distance of 5.0×10^{-11} m.

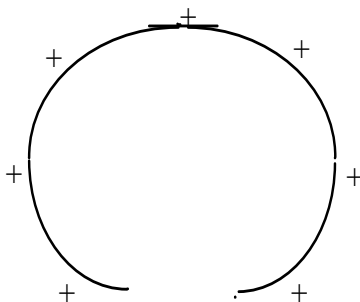
Calculate the potential energy associated with an electron in a hydrogen atom.

- Consider an equilateral triangle PQR where $QR = 20$ mm. A charge of $+1.0 \times 10^{-8}$ C is placed at Q and a charge of -1.0×10^{-8} C is placed at R. Both charges are fixed in place.
 - Calculate the electric field strength at point P.
 - Calculate the electrostatic potential at point P.
- The diagram below shows two horizontal metal plates X and Y which are separated by a distance of 50 mm.. There is a potential difference between the plates of 1200 V. Note that the lower plate, X, is earthed.



- Draw a sketch graph to show how the potential varies along a line joining the mid-point of plate X to the mid-point of plate Y.
- Calculate the electric field strength between the plates.
- Explain how the value for the electric field strength can be obtained from the graph obtained in (a).

- 6 (a) State what is meant by an equipotential surface.
- (b) The sketch below shows the outline of the positively charged dome of a Van de Graaff generator.



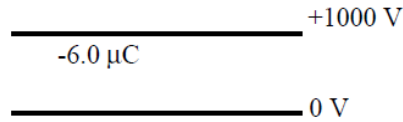
Copy this sketch and show the electric field lines and equipotential surfaces around the charged dome.

- 7 Two oppositely charged parallel plates have a potential difference of 1500 V between them. If the plates are separated by a distance of 20 mm calculate the electric field strength, in V m^{-1} , between the plates.
- 8 A uniform electric field is set up between two oppositely charged parallel metal plates by connecting them to a 2000 V d.c. supply. The plates are 0.15 m apart.
- (a) Calculate the electric field strength between the plates.
- (b) An electron is released from the negative plate.
- (i) State the energy change which takes place as the electron moves from the negative to the positive plate.
- (ii) Calculate the work done on the electron by the electric field.
- (iii) Using your answer to (ii) above calculate the velocity of the electron as it reaches the positive plate.
- 9 A proton is now used in the **same** electric field as question 8 above. The proton is released from the positive plate.
- (a) Describe the motion of the proton as it moves towards the negative plate.
- (b) (i) Describe how the work done on the proton by the electric field compares with the work done on the electron in question 8.
- (ii) How does the velocity of the proton just as it reaches the negative plate compare with the velocity of the electron as it reaches the positive plate in the previous question?
- 10 (a) A sphere of radius 0.05 m has a potential at its surface of 1000 V. Make a sketch of the first 5 equipotential lines outside the sphere if there is 100 V between the lines. (i.e. calculate the various radii for these potentials).
- (b) Calculate the charge on the sphere.

- 11 (a) Using the equation $v = \sqrt{\frac{2eV}{m}}$, calculate the speed of an electron which has been accelerated through a potential difference of 1.0×10^6 V.
- (b) Is there anything wrong with your answer?

Charges in motion 1

1. Two parallel plates are connected to a 1000 V supply as shown below. A $-6.0 \mu\text{C}$ charge is just at the lower surface of the top plate.

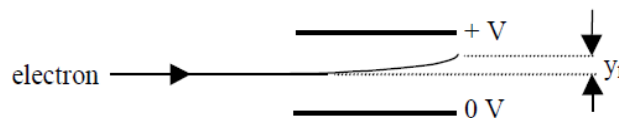


- (a) How much work is done in moving the $-6.0 \mu\text{C}$ charge between the plates?
- (b) Describe the energy transformation associated with the movement of a $-6.0 \mu\text{C}$ charge, when it is released from the **bottom** plate.
2. A p.d. of $3.0 \times 10^4 \text{ V}$ is applied between two large parallel plates. The electric field strength between the plates is $5.0 \times 10^5 \text{ N C}^{-1}$.

- (a) Determine the separation of the parallel plates.
- (b) The separation of the plates is reduced to half the value found in (a). What will happen to the magnitude of the electric field strength between the plates?
- (c) An electron leaves one plate from rest and is accelerated towards the positive plate. Show that the velocity of the electron just before it reaches the positive plate is given by

$$v = \sqrt{\frac{2Ve}{m}} \quad \text{where } V \text{ is the p.d. between the plates.}$$

3. An electron is projected along the axis midway between two parallel plates as shown below.



The kinetic energy of the electron is $2.88 \times 10^{-16} \text{ J}$.

The magnitude of the electric field strength between the plates is $1.4 \times 10^4 \text{ N C}^{-1}$.

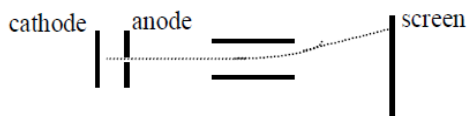
The length of the plates is 0.15 m. The plate separation is 0.10 m.

- (a) Determine the initial horizontal speed of the electron as it enters the space between the plates.
- (b) What is the vertical deflection, y_1 , of the electron?
- (c) Describe the motion of the electron after it leaves the space between the plates.

4. A beam of electrons is accelerated from rest at a cathode towards an anode. After passing through the hole in the anode the beam enters the electric field between two horizontal plates as shown below.

A screen is placed 0.180 m beyond the end of the plates.

You may assume that there is no electric field between the anode and parallel plates and no electric field between the parallel plates and screen.



- (a) The p.d. between the cathode and anode is 200 V.
- Calculate the speed of each electron as it enters the space between the plates.
- (b) The p.d. across the plates is 1.0 kV. The plates are 30 mm long and their separation is 50 mm. Calculate the deflection of an electron on leaving the parallel plates.
- (c) Calculate the total deflection on the screen.
5. Electrons are accelerated through a p.d. of 125 kV.
- (a) What speed would this give for the electrons, assuming that $qV = \frac{1}{2}mv^2$?
- (b) Why is the answer obtained in (a) unlikely to give the correct speed for the electrons?
6. Explain how the results of Millikan's experiment lead to the idea of quantisation of charge.
7. In a Millikan oil drop experiment the oil drop has a mass of $0.01 \mu\text{g}$. It is suspended between two plates that are 20 mm apart. The charge on the drop is found to be $-5e$.
- (a) Draw a sketch of the drop showing the forces acting on the drop. The upthrust of the air may be neglected.
- (b) Determine the p.d. between the plates.
- (c) The p.d. between the plates is increased.
- Describe and explain what would happen to the drop?

8. In a Millikan type experiment, a small charged oil drop is held stationary between two plates by adjusting the p.d. between the plates. The experiment is repeated a number of times with different oil drops. The readings below show the mass of each oil drop and the p.d. required to hold it stationary.

Mass of drop/ 10^{-15} kg	2.6	1.2	1.6	2.3	4.8	5.9	1.8	3.7
p.d. / V	1592	2940	1960	2818	2940	14455	1470	4533

The plate separation is 40 mm.

- (a) For each set of readings calculate the number of excess electrons on the oil drop.
- (b) Suggest why these readings indicate that charge is quantised.
9. An alpha particle is about to make a head on collision with an oxygen nucleus. When at a large distance from the oxygen nucleus, the speed of the alpha particle was $1.9 \times 10^6 \text{ m s}^{-1}$. The mass of the alpha particle is $6.7 \times 10^{-27} \text{ kg}$.
- (a) State an expression for the change in kinetic energy of the alpha particle as it approaches the oxygen nucleus.
- (b) State an expression for the change in electrostatic potential energy of the alpha particle.
- (c) Using your answers to (a) and (b) derive an expression for the distance, r , of closest approach.
- (d) Calculate the distance of closest approach for the alpha particle to the oxygen nucleus.
10. The distance of closest approach between an alpha particle and an iron nucleus is $1.65 \times 10^{-13} \text{ m}$. The mass of an alpha particle is $6.7 \times 10^{-27} \text{ kg}$ and the atomic number of iron is 26.

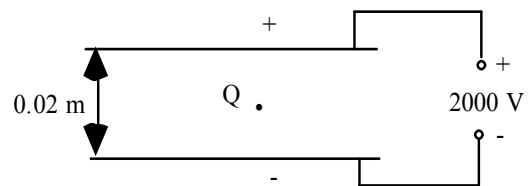
What was the speed of approach of the alpha particle?

Charges in Motion 2

- 1 (a) Calculate the acceleration of an electron in a uniform electric field of strength $1.2 \times 10^6 \text{ V m}^{-1}$.
- (b) An electron is accelerated from rest in this electric field.
- (i) How long would it take for the electron to reach a speed of $3.0 \times 10^7 \text{ m s}^{-1}$?
- (ii) Calculate the displacement of the electron in this time
- 2 In a Millikan type experiment a very small oil drop is held stationary between the two plates.

The mass of the oil drop is $4.9 \times 10^{-15} \text{ kg}$.

- (a) State the sign of the charge on the oil drop.
- (b) Calculate the size of the charge on the oil drop.



- 3 An α -particle travels at a speed of $5.0 \times 10^6 \text{ m s}^{-1}$ in a vacuum.
- (a) Calculate the minimum size of electric field strength necessary to bring the α -particle to rest in a distance of $6.0 \times 10^{-2} \text{ m}$.
(The mass of an α -particle is $6.7 \times 10^{-27} \text{ kg}$).
- (b) Draw a sketch of the apparatus which could be used to stop an α -particle in the way described above.
- (c) Can a γ -ray be stopped by an electric field? Explain your answer.
- 4 In an oscilloscope an electron enters the electric field between two horizontal metal plates. The electron enters the electric field at a point midway between the plates in a direction parallel to the plates. The speed of the electron as it enters the electric field is $6.0 \times 10^6 \text{ m s}^{-1}$. The electric field strength between the plates is $4.0 \times 10^2 \text{ V m}^{-1}$.
- The length of the plates is $5.0 \times 10^{-2} \text{ m}$. The oscilloscope screen is a **further** 0.20 m beyond the plates.
- (a) Calculate the time the electron takes to pass between the plates.
- (b) Calculate the vertical displacement of the electron on leaving the plates.
- (c) Calculate the final direction of the electron on leaving the plates.
- (d) What is the total vertical displacement of the electron on hitting the oscilloscope screen.

- 5 Electrons are accelerated through a large potential difference of 7.5×10^5 V. The electrons are initially at rest.

Calculate the speed reached by these electrons.

- 6 In the Rutherford scattering experiment α -particles are fired at very thin gold foil in a vacuum. On very rare occasions an α -particle is observed to rebound back along its incident path.

(a) Explain why this is not observed very often.

(b) The α -particles have a typical speed of 2.0×10^7 m s⁻¹.

Calculate the closest distance of approach which an α -particle could make towards a gold nucleus in a head-on collision. The atomic number of gold is 79. The mass of the α -particle is 6.7×10^{-27} kg.

- 7 In Millikan's experiment, an negatively charged oil drop of radius 1.62×10^{-6} m is held stationary when placed in an electric field of strength 1.9×10^5 V m⁻¹.

(The density of the oil is 870 kg m⁻³, and the volume of a sphere is $\frac{4}{3} \pi r^3$.)

(a) Calculate the mass of the oil drop.

(b) Calculate the charge on the oil drop.

(c) How many electronic charges is your answer to (b) equivalent to?

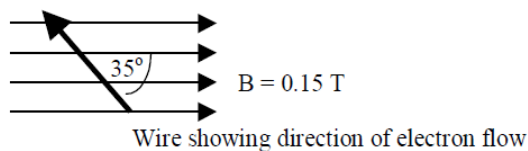
- 8 A charged particle has a charge-to-mass ratio of 1.8×10^{11} C kg⁻¹.

Calculate the speed of one such particle when it has been accelerated through a potential difference of 250 V.

Electromagnetism

- State the condition for a magnetic field to exist.
 - Under what conditions will a charged particle experience a force in a magnetic field?
 - State the definition of the tesla.
- State the expression for the force on a current carrying conductor placed at an angle θ in a magnetic field.
 - Draw a sketch to show the position of this angle θ , the direction of the electron flow in the conductor, the direction of the magnetic induction and the direction of the force.
 - A straight conductor of length 25 mm carries a current of 2.0 A. It experiences a force of 9.5 mN when placed in a magnetic field with a magnetic induction of 0.70 T.
Calculate the angle between the direction of the magnetic field and the conductor.
- A wire, carrying a current of 10 A, is placed at right angles to a magnetic field. A straight section of the wire 0.80 m long has a force of 0.20 N acting on it.

Calculate the size of the magnetic induction of the magnetic field.
- A straight wire of length 0.50 m is placed in a region of magnetic induction 0.10 T.
 - What is the minimum current required in the wire to produce a force of 0.30 N on the wire?
 - Why is this a minimum value?
- A wire of length 200 mm is placed at an angle of 35° to a magnetic field of magnetic induction 0.15 T.



- The current in the wire is 7.0 A.

Calculate the magnitude of the force on the wire.
 - State the direction of this force.
- State the expression for the magnetic induction at a perpendicular distance r from an infinite straight conductor carrying a current I .
 - Derive the expression

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

using the expression stated in question 6. State clearly the meaning of all the symbols in this expression.

8. Two long parallel wires are placed 90 mm apart in air. One of the wires is carrying a current of 2.0 A and the force per unit length on the wire is $8.89 \times 10^{-6} \text{ N m}^{-1}$. What is the current in the other wire?
9. A long wire X is fixed horizontally to the ground. A second very thin wire, Y, of weight 0.075 newtons per metre length, runs parallel to wire X. The magnetic repulsion between the wires causes wire Y to be suspended 5.0 mm above wire X. The wires carry the same current, I. Calculate the value of I.

Force on a Conductor

- 1 A straight wire, 0.05 m long, is placed in a uniform magnetic field of magnetic induction 0.04 T.

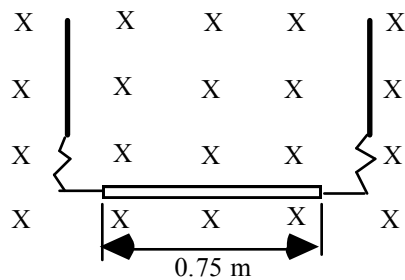
The wire carries a current of 7.5 A, and makes an angle of 60° with the direction of the magnetic field.

- Calculate the magnitude of the force exerted on the wire.
- Draw a sketch of the wire in the magnetic field and show the direction of the force,
- Describe the conditions for this force to be a maximum.

- 2 A straight conductor of length 50 mm carries a current of 1.4 A. The conductor experiences a force of 4.5×10^{-3} N when placed in a uniform magnetic field of magnetic induction 90 mT.

Calculate the angle between the conductor and the direction of the magnetic field.

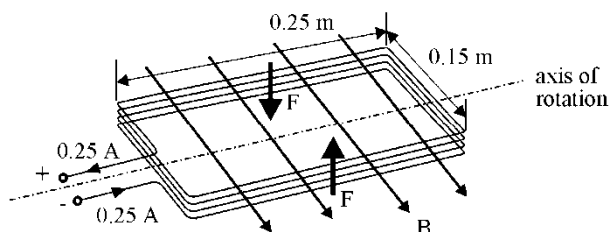
- 3 A wire of length 0.75 m and mass 0.025 kg is suspended from two very flexible leads as shown.



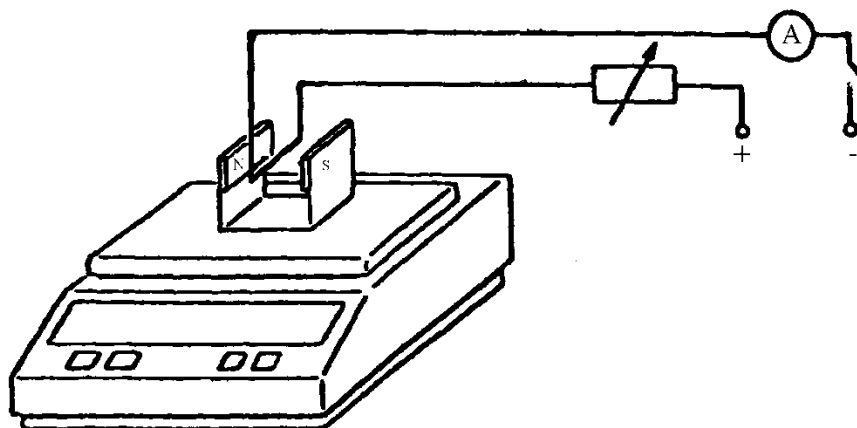
wire is in a magnetic field of magnetic induction 0.50 T. The magnetic field direction is 'into the page'.

- Calculate the size of the current in the wire necessary to remove the tension in the supporting leads.
- Copy the sketch and show the direction of the electron current which produced this result.

- 4 The sketch below shows the rectangular coil of an electric motor. The coil has 120 turns and is 0.25 m long and 0.15 m wide and carries a current of 0.25 A. It lies parallel to a magnetic field of magnetic induction 0.40 T. The sketch shows the directions of the forces acting on the coil.



- (a) Calculate the magnitude of the force, F , on each of the wires shown.
- (b) Calculate the torque which acts on the coil when in this position.
- (c) State and explain what will happen to this torque as the coil starts to rotate in the magnetic field.
- 5 The diagram below shows a force-on-a-conductor balance set up to measure the magnetic induction between two flat magnets in which a north pole is facing a south pole.



The length of the wire in the magnetic field is 0.06 m.

When the current in the wire is zero, the reading on the balance is 95.6 g. When the current is 4.0 A the reading on the balance is 93.2 g.

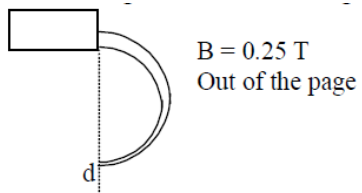
- (a) Calculate the magnitude and direction of the force on the wire from these balance readings.
- (b) Calculate the size of the magnetic induction between the poles of the magnets.
- (c) Suggest what the reading on the balance would be if the connections to the wire from the supply were reversed. Explain your answer.
- (d) Suggest what the reading on the balance would be if one of the magnets is turned over so that the north face on one magnet is directly opposite the north face of the other magnet. Explain your reasoning.

- 6 Two parallel wires 0.20 m apart carry large direct currents to an iron recycling plant. The large currents passing into the metal generate enough heat to make the iron melt. It can then be made into new shapes.
- (a) Calculate the force between two such wires 16 m long if they each carry a current of 2500 A.
 - (b) Hence explain why these wires are not suspended freely on their route to the iron smelter.

Motion in a magnetic field

1. Derive the expression $F = qvB$ using the relationship $F = I/B\sin\theta$ with $\theta = 90^\circ$.
State clearly the name and unit of all the symbols in this expression.
2. A proton travels at right angles to a magnetic field of magnetic induction 0.80 T.
The speed of the proton is $3.0 \times 10^4 \text{ m s}^{-1}$. Determine the force on the proton.
3. An electron is moving at right angles to a magnetic field of magnetic induction 0.50 T. The velocity of the electron is $2.0 \times 10^5 \text{ m s}^{-1}$.
 - (a) Calculate the magnitude of the force on the electron.
 - (b) State the direction of the force on the electron.
 - (c) Determine the radius of the circular path of the electron.
4. The movement of an electron in a uniform magnetic field is found to be helical.
Explain how this helical movement arises.
5. 'Crossed' electric and magnetic fields can be used in a velocity selector.
 - (a) Explain what is meant by 'crossed' electric and magnetic fields.
 - (b) The velocity selector 'selects' charged particles, which pass through the fields without being deflected. By considering the magnetic force and electrostatic force on a charged particle show that the 'selected' velocity is E/B .
 - (c) State, with a reason, if the selected velocity depends on:
 - (i) the charge of the particle;
 - (ii) the mass of the particle.
 - (d) In a mass spectrometer ions from a velocity selector enter a region that only has a magnetic field. With the aid of a sketch, explain how the ions can be identified by their deflection.
6. In a J J Thomson type experiment the charge to mass ratio is to be determined. Crossed magnetic and electric fields are used to produce an undeflected beam.
 - (a) Derive an expression for the velocity of the electrons in this undeflected beam in terms of the magnetic induction, B , the p.d. across the plates, V , and the plate separation, d .
 - (b) The magnetic field is then applied by itself and the electron beam moves in a circular path of radius, r .
By considering the central force on the electrons derive an expression for e/m in terms of the velocity of the electrons and this radius, r .
 - (c) Use the expressions stated in (a) and (b) above to show that
$$\frac{e}{m} = \frac{V}{rB^2d}$$

7. In a mass spectrometer two isotopes of single ionised carbon-13 and carbon-12 ions are accelerated by a p.d. of 4.0 kV. They emerge from a small slit into a uniform magnetic field of magnetic induction 0.25 T as shown below.



Mass of carbon-13 ion = 2.16×10^{-26} kg

Mass of carbon-12 ion = 1.99×10^{-26} kg

Calculate the separation, d , of the two carbon ions.

Charged Particles in Magnetic Fields

- 1 A particle carrying a positive charge of $1.6 \times 10^{-19} \text{ C}$ travels at $1.0 \times 10^7 \text{ m s}^{-1}$ and enters a magnetic field at an angle of 45° . The force experienced by the particle is $3.0 \times 10^{-17} \text{ N}$. Calculate the size of the magnetic induction required to produce this result.
- 2 Close to the equator the horizontal component of the Earth's magnetic field has a magnetic induction of $3.3 \times 10^{-5} \text{ T}$. A high energy proton arrives from outer space with a vertical velocity of $2.8 \times 10^8 \text{ m s}^{-1}$.

Show that the ratio of the magnetic force, F_m , to gravitational force, F_g , experienced by the proton is given by $\frac{F_m}{F_g} = 3.2 \times 10^{10}$.

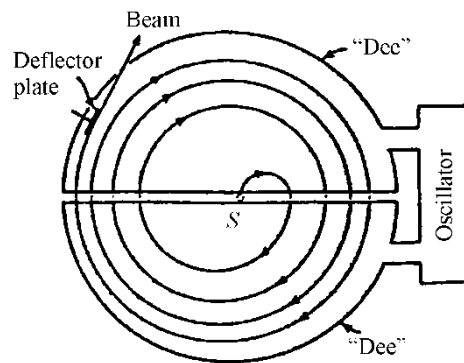
- 3 Inside a bubble chamber, a proton is injected at right angles to the direction of the magnetic field of magnetic induction 0.28 T . The kinetic energy of the proton is $4.2 \times 10^{-12} \text{ J}$.

Calculate the radius of the circle described by the track of the proton.

- 4 An electron and an alpha particle both make circular orbits in the same magnetic field. Both particles have the same orbital speed. The mass of an alpha particle is $6.68 \times 10^{-27} \text{ kg}$.

Calculate the ratio $\frac{\text{frequency of rotation of electrons}}{\text{frequency of rotation of alpha particles}}$.

- 5 A cyclotron has an oscillator frequency of $1.2 \times 10^7 \text{ Hz}$ and a maximum effective dee radius of 0.50 m . The sketch below shows the geometry of the cyclotron. The deflector plate is mounted at the maximum radius of 0.50 m and its purpose is to ensure that the charged particles exit successfully.



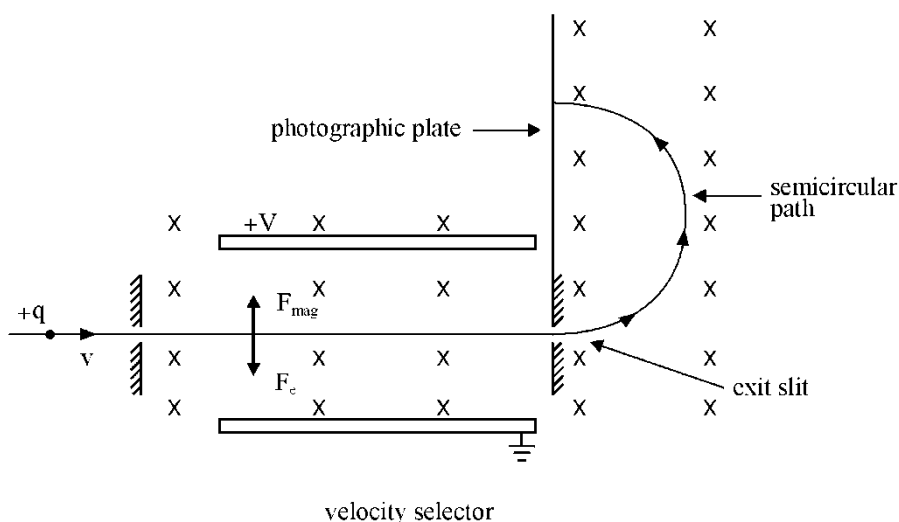
- (a) Show that the frequency of rotation is given by $f = \frac{qB}{2\pi m}$.
- (b) The cyclotron is used to accelerate deuterons from rest. Calculate the magnetic induction, of the magnetic field of the cyclotron, needed to accelerate the deuterons. (A deuteron is an isotope of hydrogen, containing a proton and a neutron, with a mass of $3.34 \times 10^{-27} \text{ kg}$.)
- (c) Calculate the kinetic energy of the emerging deuterons.

- 6 (a) Certain charged particles can be 'selected' from a beam of charged particles if they are made to enter crossed electric and magnetic fields. The directions of the fields are mutually perpendicular. Show that those particles with a velocity equal to the ratio $\frac{E}{B}$ will be undeflected.

- (b) A velocity selector such as that in (a) above has a magnetic induction of 0.70 T and an electric field strength of $1.4 \times 10^5 \text{ V m}^{-1}$.

Calculate the velocity of the undeflected charged particles which pass through the crossed fields.

- (c) The sketch below shows a mass spectrometer arrangement used to deflect ions. After the exit slit from the velocity selector there is only a magnetic field present. (The final position of the ions is detected by a photographic plate).



- (i) Negatively charged neon ions, which carry one extra electron, emerge from the velocity selector into a uniform magnetic field of magnetic induction 0.70 T. The ions follow the semicircular path shown above. The radius of the circle is 70 mm.

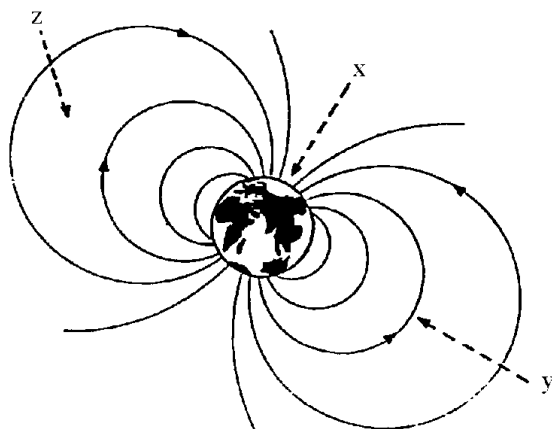
Calculate the mass of the neon ions.

- (ii) Isotopes are nuclei having the same atomic number but different mass numbers. Explain how the mass spectrometer can show the presence of different isotopes of neon.

- 7 An alpha particle travels in a circular orbit of radius 0.45 m while moving through a magnetic field of magnetic induction 1.2 T. Calculate:

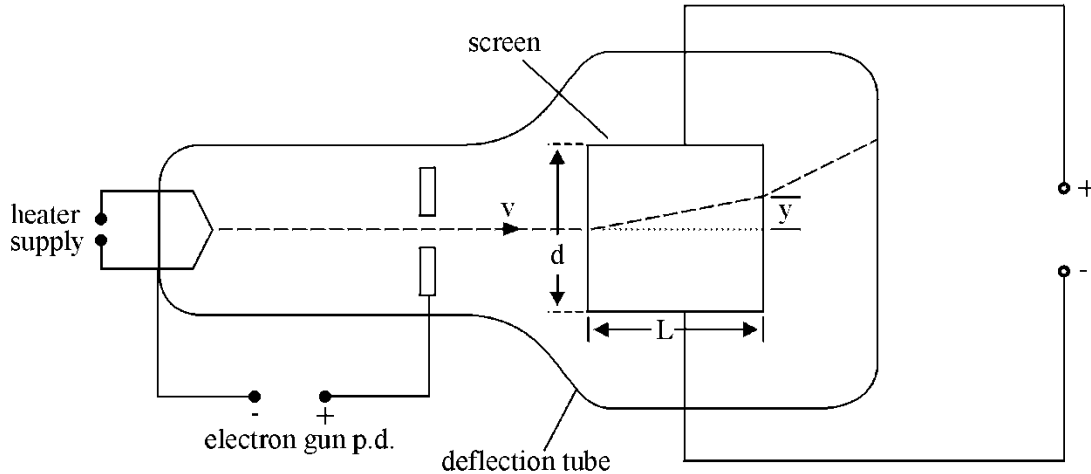
- (a) the speed of the alpha particle in its orbit
 (b) the orbital period of the alpha particle
 (c) the kinetic energy of the alpha particle in its orbit.

- 8 The diagram below shows the Earth's magnetic field. Three positively charged particles, X, Y and Z approach the Earth along the directions shown. Path Y is perpendicular to the direction of the Earth's magnetic field lines.



- (a) For each of the particles, describe the path followed in the Earth's magnetic field.
- (b) A proton approaches the earth along path Y with a speed of $2.0 \times 10^6 \text{ m s}^{-1}$. Calculate the radius of the path of the proton at a point where the magnetic induction of the Earth is $1.3 \times 10^{-5} \text{ T}$.
9. In an experiment to measure the charge to mass ratio for an electron, an electron beam is fired between parallel plates in a vacuum tube. The vacuum tube is placed between a pair of Helmholtz coils.
- (a) Crossed electric and magnetic fields are applied to produce an undeflected beam. The current, I , in the Helmholtz coils is measured to be 0.31 A . The p.d., V , across the plates is 1200 V .
- (i) Show that the velocity, v , of the electrons between the plates is given by $v = \frac{V}{Bd}$ where d is the plate separation and B the magnetic induction.
- (ii) The magnetic induction, B , is given by $\frac{9 \times 10^{-7} NI}{a}$ where N , the number of turns, is 320 and a , the effective radius of the coils, is 0.073 m . Calculate the magnetic induction B .
- (iii) The plate separation, d , is 0.045 m . Calculate the magnitude of the velocity of the electrons.
- (b) The electric field is switched off and the above magnetic field only is applied. A deflection, y , of 0.015 m is measured.
- (i) Show that the charge to mass ratio $\frac{e}{m} = \frac{v}{rB}$ where r is the radius of the curved path of the electrons when only the magnetic field is applied.
- (ii) The radius, r , is given by $r = \frac{L^2 + y^2}{2y}$ where L , the length of the plates, is 0.055 m . Calculate the charge to mass ratio for the electrons from this data.

- 10 The sketch below shows the layout of the apparatus which allows crossed electric and magnetic fields to be applied to an electric beam. (The Helmholtz coils which produce the magnetic field 'into the paper' have been omitted for clarity).



The screen is 0.05 m square. The length, L , and separation, d , are both 0.05 m.

- (a) With only the electric field switched on the p.d. between the plates is set at 1200 V. The vertical deflection, y , of the electron beam at the end of the plates is 1.0 cm. Calculate the electric field strength between the plates which provides this deflection.
- (b) (i) **Both** the electric and magnetic fields are now applied. The electron beam is undeflected when the current to the Helmholtz coils is 0.25 A. The coils have 320 turns and an effective radius of 0.068 m.

Using the expression $B = \frac{8m_0NI}{\sqrt{125} r}$, calculate the value this gives for the magnetic induction between the plates.

- (ii) Calculate the speed of the electrons as they enter the plates.
- (c) With the electric field **only** applied, the electron beam has a vertical deflection, y of 1.0 cm at the end of the plates.

(i) Show that the deflection $y = \frac{1}{2} a \frac{L^2}{v^2}$

where a is the acceleration produced by the electric field and v is the speed of the electrons entering the plates as shown in the diagram.

- (ii) Using the data in the question, calculate a value for the charge to mass ratio, $\frac{e}{m}$, for the electron.

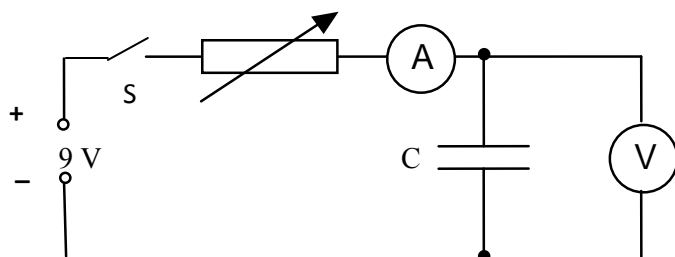
11. Apparatus similar to that shown in question 10 is used in another experiment to determine the charge to mass ratio for the electron. The length of the plates L and their separation d are 0.05 m.

In **this** experiment the p.d. across the electron gun is set at 1000 V. Assuming the electrons leave the cathode with zero speed, the speed of the electrons entering the plates can be determined using this electron gun potential difference.

- (a) Show that $\frac{e}{m} = \frac{v^2}{2 \times 1000}$ where v is the speed of the electrons as they leave the electron gun.
- (b) Both the electric and magnetic fields are applied to give an undeflected beam. The p.d. across the plates is 1000 V. The current in the Helmholtz coils to give an undeflected beam is measured to be 0.26 A.
- (i) Use the expression $B = \frac{9 \times 10^{-7} NI}{r}$ where the number of turns, N , is 320 and the effective radius r of the coils is 0.068 m, to calculate the size of the magnetic induction between the plates.
- (ii) Calculate the magnitude of the speed v of the undeflected electrons when they are between the plates.
- (c) Calculate the charge to mass ratio for the electrons from this data.
- (d) The overall uncertainty in this experiment was estimated to be 5 %.
- (i) Express the measured value of the charge to mass ratio as (value \pm absolute uncertainty).
- (ii) Consider the accepted value and comment on the accuracy of this experiment.

Capacitors in d.c. circuits

- A $50 \mu\text{F}$ capacitor is charged until the p.d. across it is 100 V .
 - Calculate the charge on the capacitor when the p.d. across it is 100 V .
 - The capacitor is now 'fully' discharged in a time of 4.0 ms .
Calculate the average current during this time.
 - Why is this average current?
- A capacitor stores a charge of $3.0 \times 10^{-4} \text{ C}$ when the p.d. across its terminals is 600 V .
What is the capacitance of the capacitor?
- A $15 \mu\text{F}$ capacitor is charged using a 1.5 V battery.
Calculate the charge stored on the capacitor when it is fully charged.
- A capacitor stores a charge of $1.2 \times 10^{-5} \text{ C}$ when there is a p.d. of 12 V across it. Calculate the capacitance of the capacitor.
 - A $0.10 \mu\text{F}$ capacitor is connected to an 8.0 V d.c. supply. Calculate the charge stored on the capacitor when it is fully charged.
- In the circuit below the capacitor C is initially uncharged.



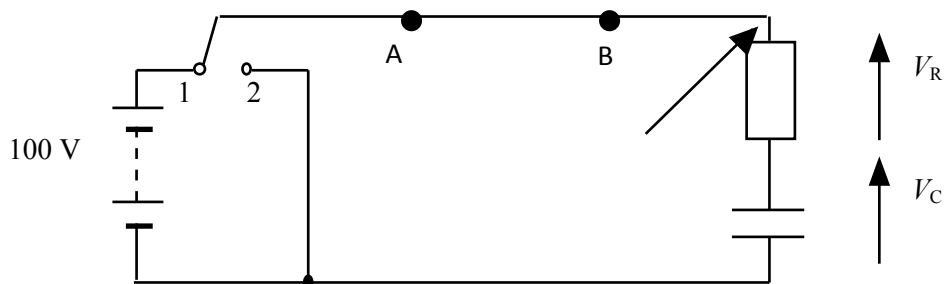
Switch S is now closed. By carefully adjusting the variable resistor R a constant charging current of 1.0 mA is maintained.

The reading on the voltmeter is recorded every 10 s . The results are shown in the table.

Time / s	0	10	20	30	40
V / V	0	1.9	4.0	6.2	8.1

- Plot a graph of the charge on the capacitor against the p.d. across the capacitor.
- Use the graph to calculate the capacitance of the capacitor.

6. The circuit below is used to charge and discharge a capacitor.



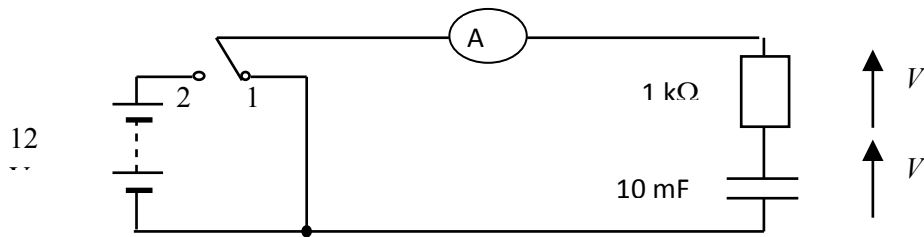
The battery has negligible internal resistance.

The capacitor is initially uncharged.

V_R is the p.d. across the variable resistor and V_C is the p.d. across the capacitor.

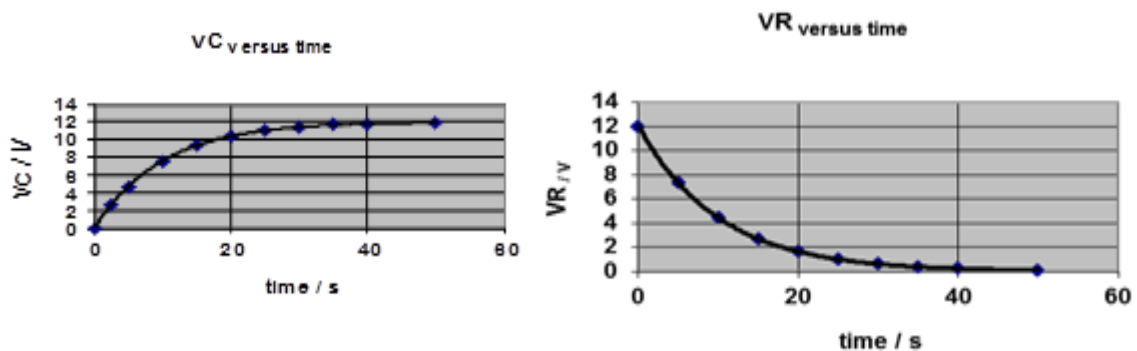
- (a) Is the position of the switch at 1 or 2
- in order to charge the capacitor
 - in order to discharge the capacitor?
- (b) Sketch graphs of V_R against time for the capacitor charging and discharging. Show numerical values for the maximum and minimum values of V_R .
- (c) Sketch graphs of V_C against time for the capacitor charging and discharging. Show numerical values for the maximum and minimum values of V_C .
- (d) (i) When the capacitor is charging what is the direction of travel of the electrons between points A and B in the wire?
- (ii) When the capacitor is discharging what is the direction of travel of the electrons between points A and B in the wire?
- (e) The capacitor has a capacitance of $4.0 \mu\text{F}$. The resistor has resistance of $2.5 \text{ M}\Omega$.
- Calculate:
- the maximum value of the charging current
 - the charge stored by the capacitor when the capacitor is fully charged.

7. The circuit shown is used to investigate the charge and discharge of a capacitor.



The switch is in position 1 and the capacitor is uncharged.

The switch is now moved to position 2 and the capacitor charges.



s show how V_C , the p.d. across the capacitor, and V_R , the p.d. across the resistor, vary with time.

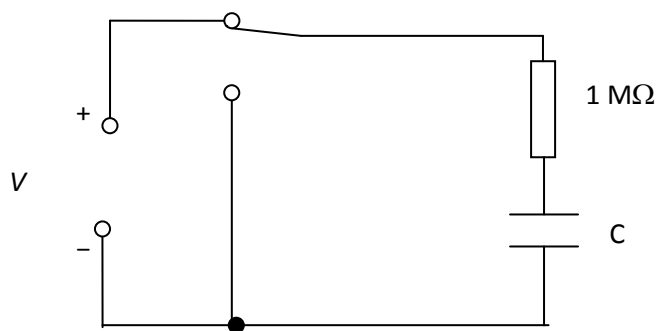
- Use these graphs to sketch a graph to show how the current varies with time in the circuit.
- The experiment is repeated with the resistance changed to $2\text{ k}\Omega$.

Sketch the graphs above and on each graph sketch the new lines which show how V_C , V_R and I vary with time.

- The experiment is repeated with the resistance again at $1\text{ k}\Omega$ but the capacitor replaced with one of capacitance 20 mF . Sketch the original graphs again and on each graph sketch the new lines which show how V_C , V_R and I vary with time.
- What does the area under the current against time graph represent?
 - Compare the areas under the current versus time graphs in part (a) and in your answers to (b) and (c). Give reasons for any increase or decrease in these areas.
- At any instant in time during the charging what should be the value of $(V_C + V_R)$?
- The original values of resistance and capacitance are now used again and the capacitor fully charged. The switch is moved to position 1 and the capacitor discharges.

Sketch graphs of V_C , V_R and I from the instant the switch is moved until the capacitor is fully discharged.

8. State what is meant by the time constant in an RC circuit.
9. In an RC circuit the time constant t is given by the relationship $t = RC$.
Show that the product RC has the unit of time.
10. A circuit is made up of a $2 \mu\text{F}$ capacitor and a $4 \text{ k}\Omega$ resistor. Calculate the capacitive time constant.
11. A student sets up a circuit to measure the capacitive time constants for three RC circuits as a capacitor discharges.



C is either a single capacitor or two capacitors in series. The table shows the resistance of R , the capacitor arrangement used and the value of the time constant.

Resistance of R	Capacitor arrangement	Time constant (s)
$1 \text{ M}\Omega$	$1 \mu\text{F}$ only	1
$1 \text{ M}\Omega$	$4 \mu\text{F}$ only	4
$1 \text{ M}\Omega$	$1 \mu\text{F}$ and $4 \mu\text{F}$ in series	0.8

Use the results in the table to show that the total capacitance C_{total} of two capacitors of capacitance C_1 and C_2 in series is given by

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

12. A circuit comprises a resistor of resistance R and capacitor of capacitance C connected in series. The capacitor is fully charged then discharged. The p.d. across the capacitor as it discharges is given by $V = V_0 e^{-t/RC}$.

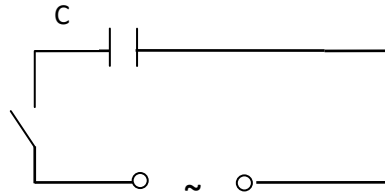
where V_0 is the p.d. across the capacitor when fully charged.

Show that at a time equal to the capacitive time constant RC , after the capacitor starts to discharge, the p.d. across the capacitor will be given by $V = 0.37V_0$.

- (b) A $4.0 \mu\text{F}$ capacitor is charged to a p.d. of 12 V . It is then connected across a $2.0 \text{ M}\Omega$ resistor so that it discharges.
- Calculate the capacitive time constant.
 - Calculate the p.d. across the capacitor 4 s after it starts to discharge.

Capacitors in a.c. circuits

1. A capacitor is connected to a variable frequency a.c. supply as shown below. The amplitude of the output voltage from the supply is kept constant.



Variable frequency

- (a) The capacitor has reactance. State what is meant by the term 'reactance'.
(b) The frequency of the output from the a.c. supply is increased.

Sketch a graph to show how:

- (i) the reactance of the capacitor varies with the frequency of the supply
(ii) the current in the circuit varies with the frequency of the supply.

2. A $1.0 \mu\text{F}$ capacitor is connected to 5.0 V a.c. power supply. The frequency of the a.c. supply is 50 Hz .
- (a) Calculate the capacitive reactance of the capacitor.
(b) Calculate the current in the circuit.
3. A capacitor is connected across a 250 V r.m.s supply having a frequency of 50 Hz . The current in the capacitor is 0.50 A r.m.s.

Calculate:

- (a) the reactance of the capacitor at this frequency
(b) the capacitance of the capacitor.
4. A 500Ω resistor and a capacitor are connected in series across an a.c. supply. The frequency of the a.c. is 50 Hz . The p.d. across the resistor is 120 V . The p.d. across the capacitor is 160 V .
- (a) Calculate the current in the circuit.
(b) Calculate the capacitance of the capacitor.

5. A 300 Ω resistor and a capacitor are connected in series with an a.c. supply of frequency 100 Hz.

The p.d. across the capacitor is 5.00 V. When the frequency of the output from the supply is 100 Hz the capacitive reactance of the capacitor is 265 Ω

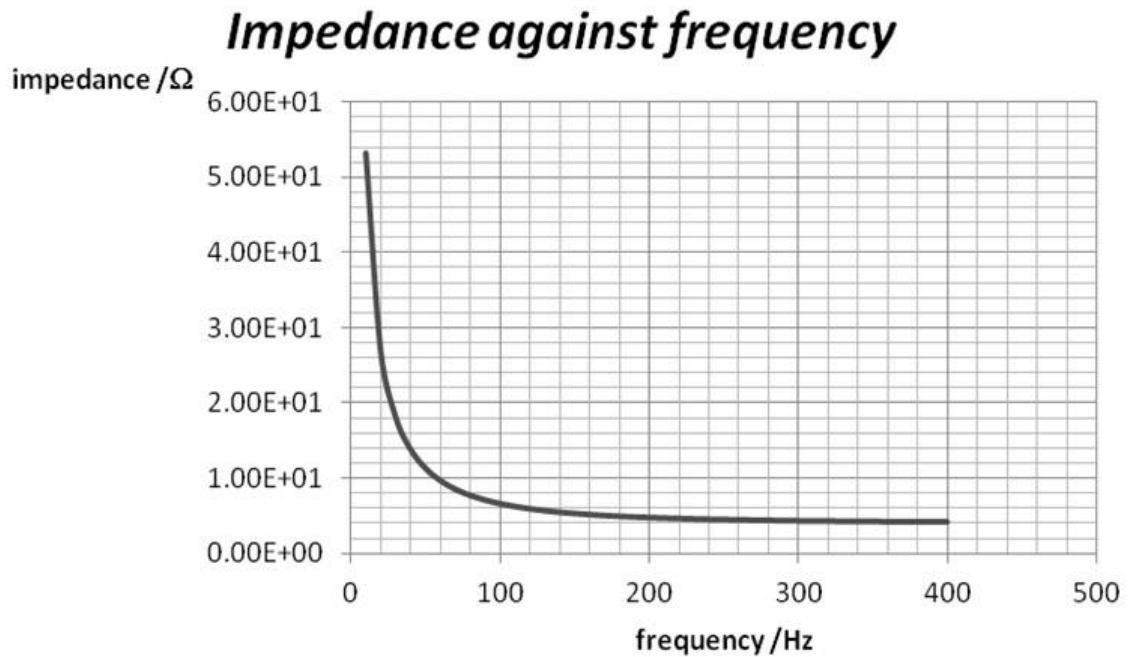
Calculate:

- the capacitance of the capacitor
 - the current in the circuit
 - the p.d. across the resistor.
6. A resistor and a capacitor are connected in series with a variable frequency constant amplitude a.c. supply. The combined effect of the resistance R and capacitive reactance X_C in series is known as the impedance Z of the circuit where

$$Z = \sqrt{R^2 + X_C^2}$$

Z can be calculated from $Z = V/I$ where V is the voltage of the supply and I is the current in the circuit.

The value of Z is found for different frequencies and the data used to plot the following graph.



Use the graph and knowledge of the relationship for Z and to estimate:

- the value of the resistance;
- the value of the capacitance.

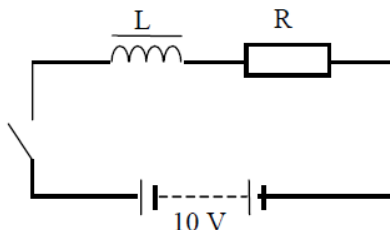
Self-inductance 1

1. (a) A student is investigating the production of an induced e.m.f. across a coil.

Describe a simple experiment which would allow her to do this.

- (b) State three ways in which the magnitude of the induced e.m.f. across a coil can be increased.

2. An inductor, resistor and battery are joined in series as shown below.



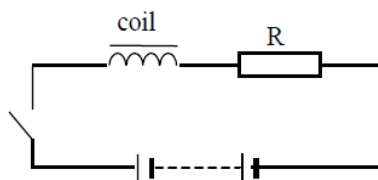
- (a) The inductor has a large number of turns. The switch is closed. Sketch a graph to show how the current in the circuit varies with time.

- (b) Explain why the current does not reach its maximum value immediately.

- (c) The resistance of the resistor is reduced. How would the shape of the graph alter ?

- (d) The number of turns on the inductor is considerably reduced. State how the graph drawn in part (a) would alter.

3. A circuit is set up as shown below.



Explain how an induced e.m.f. is produced across the coil.

4. When the current through an inductor is increasing the induced e.m.f. opposes this increase in current. The current takes time to reach its maximum value.

- (a) Explain what happens when the current through an inductor decreases.

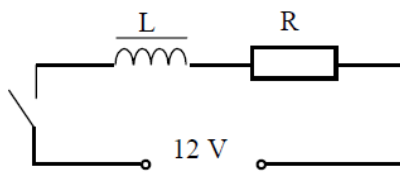
- (b) The current through an inductor decreases. Use the conservation of energy to explain the direction of the induced e.m.f.

5. (a) The current through a coil changes. State the equation for the e.m.f. induced across the coil in terms of the self-inductance of the coil.

- (b) State the unit of inductance.

- (c) State the equation for the energy stored in the magnetic field of an inductor.

6. An inductor, resistor and d.c. supply are connected in series as shown below.

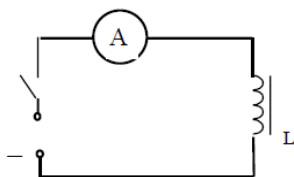


The internal resistance of the d.c. supply is negligible.

The inductance of the inductor is 0.40 H. The resistance of the resistor is 15 Ω .

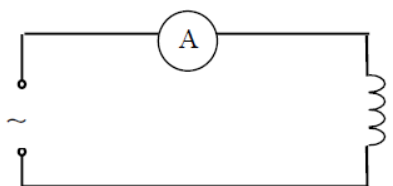
The switch is now closed.

- Why does it take a short time for the current to reach its steady value?
 - Calculate the steady current reached.
 - When the current reaches a steady value, calculate the energy stored in the inductor.
7. An inductor is connected to an ammeter and an 8.0V direct supply of negligible internal resistance, as shown below. The resistance of the inductor coil is 20 Ω .



When the reading on the ammeter is 0.10 A, the rate of change of current is 100 A s⁻¹.

- Calculate the p.d. across the coil.
 - Find the induced e.m.f. across the coil.
 - Calculate the inductance of the coil.
 - Calculate the energy stored in the inductor.
9. Which of the following are vector quantities:
induced e.m.f., self-inductance, energy stored in an inductor, rate of change of current.
10. An inductor is connected to a variable a.c. supply as shown below.



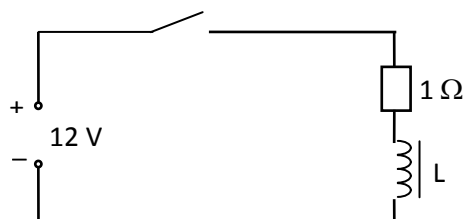
- (a) (i) The frequency of the a.c. supply is increased.
Draw a graph to show how the current in the circuit varies with the frequency of the supply.
- (ii) The inductor is removed and replaced by a capacitor. Draw another graph to show how the current in the circuit varies with the frequency of the supply.
- (b) The inductor has reactance. State what is meant by the term 'reactance'.

11. Describe an example of the use of an inductor:

- (a) as a source of a high e.m.f.
- (b) in blocking a.c. signals while transmitting d.c. signals.

Self-Inductance 2

- 1 The sketch below shows an inductor connected to a 12 V direct supply of negligible internal resistance. The resistance of the coil is 1Ω (as shown). When switched on the current grows from zero. The rate of growth is 400 A s^{-1} when the current is 8.0 A.

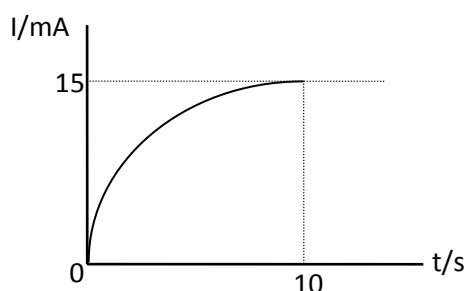
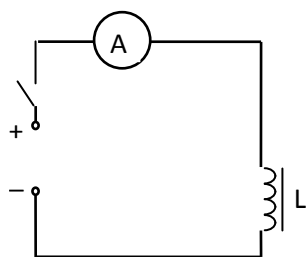


- Calculate the induced e.m.f. across the coil when the current is 8 A.
- Calculate the inductance of the coil.
- Calculate the rate of increase of current when the switch is closed.
- A final steady value of current is produced in the coil.

Find the value of this current.

- Calculate the final energy stored in the inductor.

- 2 An inductor with a removable soft iron core is connected in series with a 3.0 V direct supply of negligible internal resistance as shown below. A milliammeter is used to monitor the current in the circuit.



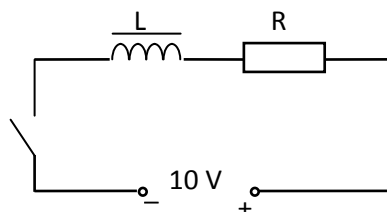
The switch is closed. The graph above shows the variation of current with time.

- Explain why it takes some time for the current to reach its maximum value.
 - Why does the current remain constant after it reaches its maximum value.
- The soft iron core is then partly removed from the coil and the experiment repeated.

Draw a sketch graph showing how the current varies against time for this second experiment. Use the same numerical values on the axes as those in the graph above.

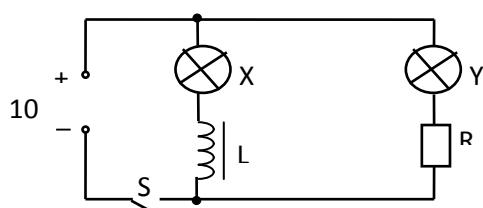
- Calculate the resistance of the coil.

- 3 In the circuit shown below, the resistance of resistor R is 40Ω and the inductance of inductor L is 2.0 H . The resistance of the inductor may be neglected. The supply has an e.m.f. of 10 V and a negligible internal resistance.



- (a) Immediately after the switch is closed:
- state the p.d. across the 40Ω resistor
 - calculate the size of the current
 - state the induced e.m.f. across the 2.0 H inductor
 - calculate the energy stored in the inductor.
- (b) Some time later the current reaches a value of 0.040 A .
- At this time, calculate the p.d. across R.
 - Calculate the p.d. across the inductor at this time.
 - Hence calculate the rate of growth of current when the current in the circuit is 0.040 A .
 - Calculate the energy stored in the inductor.

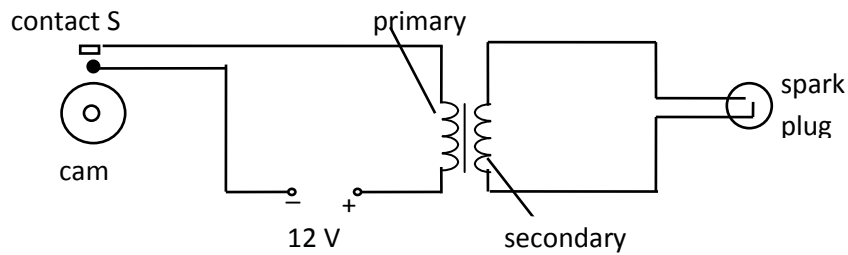
- 4 (a) Describe what is meant by the self-inductance of a coil.
- (b) The circuit diagram below shows a resistor, inductor and two lamps connected to a direct supply of 10 V . The supply has negligible internal resistance. The rating of each lamp is $6 \text{ V}, 3 \text{ W}$.



After the switch is closed each lamp operates at its rated power. However lamp Y lights up before lamp X.

- Explain why lamp Y lights before lamp X.
- The current in lamp X grows at a rate of 0.50 A s^{-1} just as the switch is closed. Calculate the inductance of the coil.
- Calculate the resistance of the coil.

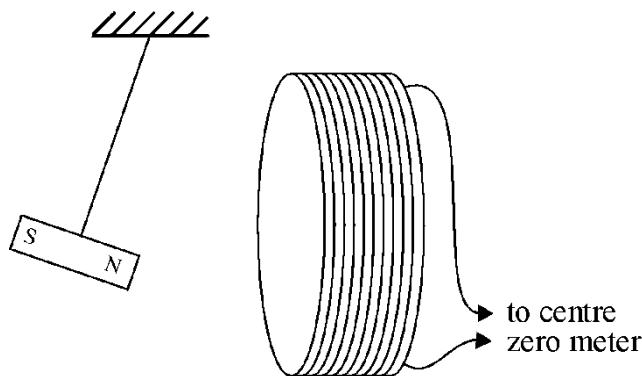
5. The diagram below shows the principle of the spark plug for a car engine.



The cam, which is rotated by the engine, makes and breaks contact at S. This switches on and off the current to the primary of a transformer.

When the contact at S is **opened** a high voltage spark of around 20 kV is induced across the spark plug.

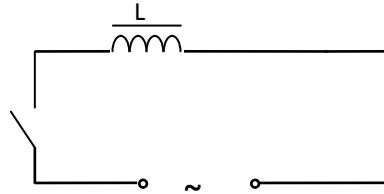
- Explain why a voltage is induced across the spark plug particularly when S is opened.
 - Explain whether a step-up or step-down transformer would be more useful in this case.
 - Where does the energy for the spark come from?
- 6 The magnet in the sketch below is mounted like a pendulum. It is allowed to swing to and fro into and out of a coil which has N turns.



- Sketch a graph to show the variation of induced e.m.f. with time as the pendulum magnet swings to and fro.
- What is the induced e.m.f. when the magnet momentarily stops?
- State what happens to the induced e.m.f. as the magnet reverses its direction of movement.
- What happens to the induced e.m.f. at the positions where the magnet moves fastest?

Inductors and a.c.

1. An inductor is connected to a variable frequency a.c. supply as shown below. The amplitude of the output voltage is kept constant.



Variable frequency

- (a) The inductor has reactance. State what is meant by the term 'reactance'.
- (b) The frequency of the a.c. supply is increased.

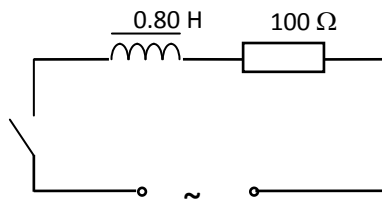
Sketch a graph to show how

- (i) the reactance of the inductor varies with the frequency of the output from the supply
- (ii) the current in the circuit varies with the frequency of the output from the supply.
2. A coil has an inductance of 0.80 H and negligible resistance. It is connected to a 30 V a.c. supply of frequency 50 Hz.
- (a) Calculate the reactance of the inductor.
- (b) Calculate the r.m.s current in the inductor.
3. A pure inductor is connected across a 250 V r.m.s supply having a frequency of 50 Hz. The current in the inductance is 0.50 A r.m.s.

Calculate:

- (a) the inductive reactance of the inductor at this frequency
- (b) the inductance of the inductor.

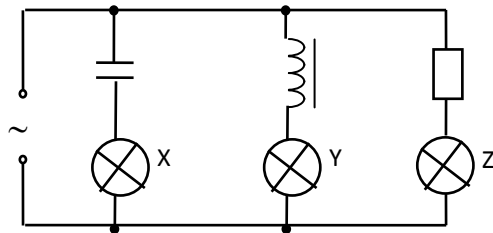
4. A pure inductor and resistor are connected across an a.c. supply with a frequency of 50 Hz.



The inductance of the inductor is 0.8 H and the resistor has a resistance of 100 Ω . The p.d. across the resistor is 12 V r.m.s.

- Calculate the current in the circuit.
- Calculate the r.m.s voltage across the inductor.

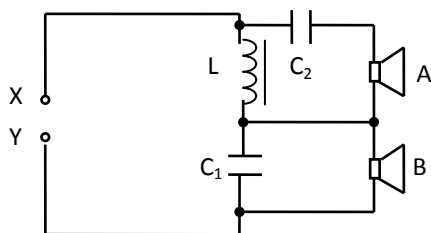
5. A circuit is set up as shown below.



The a.c. supply is of constant amplitude but variable frequency. The frequency of the supply is varied from a very low frequency to a very high frequency.

Explain what you would expect to happen to the average brightness of each of the lamps X, Y and Z as the frequency is increased.

6. The output from an amplifier is connected across XY in the circuit shown below. This is designed to direct low frequency signals to one loudspeaker and high frequency signals to the other loudspeaker.

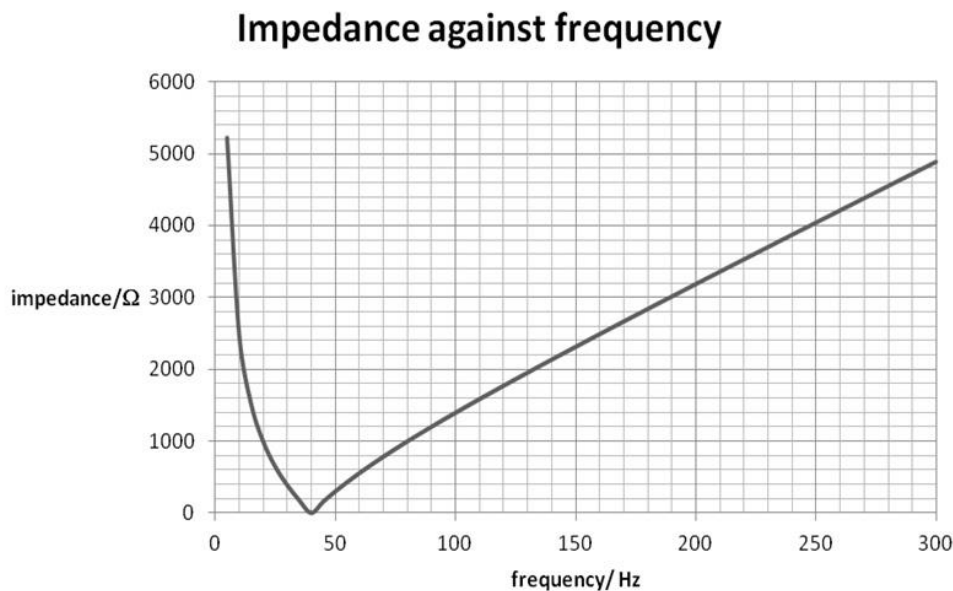


- (a) Suggest which of the loudspeakers A or B is intended to reproduce high-frequency signals.
- (b) Explain how the high- and low-frequency signals are separated by this circuit.
7. In a series circuit containing a resistor, a capacitor and an inductor the combined effect is known as the impedance Z of the circuit where

$$Z = \frac{V}{I} \quad \text{and} \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

A series circuit is made of a pure inductor and a capacitor. It is connected across an a.c. supply of constant amplitude but variable frequency.

- (a) Describe how the impedance Z of the circuit can be measured.
- (b) The measurements of Z are used to plot the following graph of Z against frequency.



- (i) Use the graph to estimate the capacitance of the capacitor.
- (ii) Use the graph to estimate the inductance of the inductor.
- (iii) The relationship shows that at a certain frequency the inductive reactance and the capacitive reactance will be equal. At this frequency the impedance will be zero. Use your results from (i) and (ii) to find this frequency and compare it to the value from the graph.

Electromagnetic radiation

1. Electromagnetic waves in a vacuum are said to be transverse. Explain the meaning of *transverse* in this context.
2. Which of the following cause/s electromagnetic radiation?
 - (a) A stationary electric charge.
 - (b) An electric charge moving with a constant acceleration.
 - (c) An accelerating electric charge.
 - (d) An electric charge in a circular particle accelerator.
 - (e) A charged particle in a linear accelerator.
 - (f) An electron in a Bohr orbit in an atom.
3. The theory of electromagnetic radiation includes the relationship

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Show that $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ has the units m s^{-1} .

4. The electric field E of an electromagnetic wave is given by

$$E = 4 \cdot 0 \times 10^2 \sin[3 \cdot 0 \times 10^6 \pi(x - 3 \cdot 0 \times 10^8 t)]$$

where E is in V m^{-1} .

Compare this relationship to that for a transverse wave

$$y = A \sin \frac{2\pi}{\lambda} (x - vt)$$

- (a) What is the amplitude of the electric field in the electromagnetic wave?
- (b) Calculate the frequency of the electric field in the electromagnetic wave.
- (c) Given that the electric field E is related to the magnetic field B by $E = cB$:
 - (i) write down the expression for the magnetic field of the electromagnetic wave
 - (ii) what is the amplitude of the magnetic field in the electromagnetic wave?

Answers

Kinematic relationships

1. (a) $90 - 8t$
(b) 11.25 s
(c) -8 m s^{-2}
2. (a) (i) -6 m s^{-1}
(ii) 0
(iii) 6 m s^{-1}
(b) 6 N
(c) Constant
3. (a) 5 m s^{-1}
(b) 11 m s^{-1}
(c) 2 m s^{-2}
(d) 1 s
4. (a) $96 \text{ m}; 86 \text{ m s}^{-1}; 54 \text{ m s}^{-2}$
(b) a depends on time t
5. (a) $v = 34.3 - 9.8t$
(b) -9.8 m s^{-2}
(c) 34.3 m s^{-1}
(d) 60 m
6. (a) $v = 30t - 100t^3; a = 30 - 300t^2$
(b) (i) 12 m
(ii) 0
(iii) 30 m s^{-2}
(c) 0 s and 0.55 s
7. (a) 1590 m s^{-1}
(b) 61200 m
(c) The rocket runs out of fuel.
9. (a) $s = 3t^2 + 2t$
(b) 6 m s^{-2}
(c) 2 m s^{-1}
10. (a) 1.67 m s^{-1}
(b) Zero
(c) -1.25 m s^{-1} (gradient of tangent to slope at 12 s)
11. (a) -3.2 m s^{-2}
(b) 4 s (or 4.1 s)
(c) Approx. 8.75 m ($17\frac{1}{2}$ boxes)

Angular motion

- 3.14 rad; 6.28 rad; 1.57 rad; 1.047 rad; 0.52 rad; 0.244 rad; 0.0174 rad
- 180° ; 360° 90° ; 57.3° ; 287° ; 5.7° ; 0.57°
- 1.6 rad s^{-1}
 - 1.57 rad s^{-1}
 - 314 rad s^{-1}
 - 251 rad s^{-1}
 - 0.105 rad s^{-1}
 - $2.67 \times 10^{-6} \text{ rad s}^{-1}$
 - $7.27 \times 10^{-5} \text{ rad s}^{-1}$
 - 5 rad s^{-1}
- 9.9 rad s^{-1}
 - 3.5 m s^{-1}
- 23.3 rad s^{-1}
 - 223 rpm
- 1.4 rad s^{-2}
 - 17.4 rad
 - 2.8 revolutions
- -6.28 rad s^{-2}
 - 112.5 revolutions
- -0.65 rad s^{-2}
 - 48 s
- 3 rad s^{-2} , -2 rad s^{-2}
 - 132 rad
 - 21 revolutions

Centripetal force and acceleration

- 189 m s^{-2}
 - 28.4 N
 - 17.6 rad s^{-1}
- 64 N
- 227 N
 - 285 N
 - 2.7 m s^{-1}
- 1643 N

- (b) $21.9 = 2.2 \times 9.8$
- (c) 23 rpm

5. 177 N

- 6. (a) Weight of mass, tension in string
- (b) Horizontal component = $T \sin 30$; vertical component = $T \cos 30$
- (c) (i) $T \cos 30$
- (c) (ii) $T \cos 30 = mg$
- (d) (i) $T \sin 30$
- (d) (ii) $T \sin 30 = mv^2/r$
- (e) 0.6 m
- (f) 1.84 m s^{-1}
- (g) 2.0 s

Moment of inertia, torque and angular acceleration

- 1. (a) 0.65 kg m^2
- (b) $1.35 \times 10^{-2} \text{ kg m}^2$
- (c) 0.36 kg m^2
- (d) 0.108 kg m^2
- (e) $4.8 \times 10^{-4} \text{ kg m}^2$
- (f) 0.045 kg m^2
- (g) 0.13 kg m^2

2. 0.61 kg m^2

- 3. (a) 4.2 N m
- (b) 3.0 N m
- (c) 63 N m

4. 1.7 N m

5. 81.8 N

- 6. (a) 0.67 rad s^{-2}
- (b) (i) 3.3 rad s^{-1}
- (b) (ii) 1.33 revolutions

7. 0.05 N m

- 8. (a) 50 s
- (b) 251 revolutions
- (c) $3.9 \times 10^3 \text{ J}$

- 9. (a) 7.7 N m (steady rotational speed, torques balanced)
- (b) 10 s (frictional torque stays constant)

10. (a) 2.4 N m

- (b) 6 rad s^{-2}
 - (c) 14 rad s^{-1}
11. (a) 9.0 N m
 (c) 46.9 rad s^{-2}
 (d) (i) 375 rad
 (ii) 188 rad s^{-1}
 (iii) $2.8 \times 10^3 \text{ J}$
 (e) 20 s
12. (b) 0.30 rad s^{-2}
 (c) (i) 0.9 rad s^{-1}
 (ii) 20.9 s

Angular momentum and rotational kinetic energy

1. (a) $3.14 \text{ kg m}^2 \text{ rad s}^{-1}$ (or units could be $\text{kg m}^2 \text{ s}^{-1}$)
 (b) 19.7 J
2. 3.2 rad s^{-1}
3. $1.5 \times 10^{-3} \text{ kg m}^2$
4. (a) 2.8 rad s^{-1}
 (b) I_{CD} will be bigger as more mass further from axis, angular velocity less.
5. 0.02 kg
6. (a) Her I decreases, so ω increases, as L is conserved.
 (b) 2.9 kg m^2
 (c) 14.7 J increase
 (d) Supplied by skater's body (chemical energy).
7. (a) 3.6 J
 (b) 3 rad s^{-1}
 (c) 1.44 J
 (d) 5.04 J
8. (a) 11.3 J
 (b) 2.24 m s^{-1}

Gravitation

1. $2.7 \times 10^{-4} \text{ N}$
3. $3.56 \times 10^{22} \text{ N}$

4. 6.02×10^{24} kg
5. (a) 3.7 N kg^{-1}
(b) 1.7 N kg^{-1}
6. (a) 9.77 N kg^{-1}
(b) 9.74 N kg^{-1}
(c) 9.73 N kg^{-1}
(d) 8.8 N kg^{-1}
7. No
8. (a) 6.65×10^6 m
(b) 90 minutes
9. 508 km
10. (a) 24 h
(b) 3.6×10^7 m
(c) $3.1 \times 10^3 \text{ m s}^{-1}$
(d) 57 N
11. 27.3 days
13. $T^2/R^3 = 1.4 \times 10^{-16}$ (constant) for all four moons
14. (a) $-6.25 \times 10^7 \text{ J kg}^{-1}$
(b) $-5.56 \times 10^7 \text{ J kg}^{-1}$
(c) $-2.7 \times 10^6 \text{ J kg}^{-1}$
15. (a) $-1.56 \times 10^{10} \text{ J}$
(b) $-3.0 \times 10^{10} \text{ J}$
(c) $-2.15 \times 10^8 \text{ J}$
16. 1.43×10^6 m
17. (a) $-1.9 \times 10^{10} \text{ J}$
(b) $9.5 \times 10^9 \text{ J}$
(c) $-9.5 \times 10^9 \text{ J}$
18. $1.6 \times 10^8 \text{ J}$ needed
21. (a) $1.12 \times 10^4 \text{ m s}^{-1}$
(b) $2.4 \times 10^3 \text{ m s}^{-1}$
(c) $5.0 \times 10^3 \text{ m s}^{-1}$
22. $1.07 \times 10^4 \text{ m s}^{-1}$
23. (a) 3.9×10^6 m

(b) $1.45 \times 10^3 \text{ m s}^{-1}$

Space and time

18. $6.67 \times 10^4 \text{ m}$

19. $1.8 \times 10^4 \text{ m}$

20. 3000 m

21. $8.9 \times 10^{-3} \text{ m}$ (8.9 mm)

22. $4.7 \times 10^3 \text{ m}$

Stellar physics

1. (a) 4410 K
(b) $2.1 \times 10^7 \text{ W m}^{-2}$

2. (a) $6.42 \times 10^7 \text{ W m}^{-2}$
(b) $3.95 \times 10^{26} \text{ W}$
(c) $1.4 \times 10^3 \text{ W m}^2$

3. (a) 12500 K
(b) $1.38 \times 10^9 \text{ J (m}^{-2} \text{ s}^{-1}\text{)}$
(c) $3.9 \times 10^{28} \text{ W}$
(d) $1.5 \times 10^9 \text{ m}$

4. $5.8 \times 10^{-9} \text{ W m}^{-2}$

5. $1.8 \times 10^{28} \text{ W m}^{-2}$

6. (a) $1.45 \times 10^{18} \text{ m}$
(b) 153 ly

7. $5.5 \times 10^{16} \text{ m}$ (5.8 ly)

8. $5.3 \times 10^{-8} \text{ W m}^{-2}$

9. Show $L = 4\pi r^2 \sigma T^4$

10. Show that apparent brightness = $\sigma r^2 T^4 / d^2$

12. 80

13. (a) 324
(b) Star A; 18 times more distant than star B.

Quantum Theory

- $\pm 1.1 \times 10^{-23} \text{ kg m s}^{-1}$
- $\pm 5.65 \times 10^{-9} \text{ m}$
- $\pm 5.3 \times 10^{-26} \text{ J}$
- $\pm 1.4 \times 10^{-26} \text{ kg m s}^{-1}$
- $1.8 \times 10^{-10} \text{ m}$
 - $6.1 \times 10^{-14} \text{ m}$
 - $2.0 \times 10^{-38} \text{ m}$
- The electron has the larger de Broglie wavelength by 1800 times.
- $4.7 \times 10^{-13} \text{ m}$
 - $1.4 \times 10^{-21} \text{ kg m s}^{-1}$
- $8.4 \times 10^6 \text{ m s}^{-1}$
 - $8.7 \times 10^{-11} \text{ m}$
 - Particle behaviour
- $2.5 \times 10^{-11} \text{ m}$ ($2.46 \times 10^{-11} \text{ m}$)
- 940 V
- $2.4 \times 10^{-11} \text{ m}$ (24 pm)
- $1.06 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$ ($1.056 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$)
 - $3.2 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$
 - Show $2\pi r = n\lambda$
 - $6.6 \times 10^{-10} \text{ m}$
 - $1.3 \times 10^{-9} \text{ m}$

Particles from space

- $5.0 \times 10^{-13} \text{ N}$
- $3.8 \times 10^{-15} \text{ N}$
- A neutron has zero charge: $q = 0$
- $6.1 \times 10^{-16} \text{ N}$ out from page
 - Zero force, as velocity is parallel to magnetic field.
- $4.5 \times 10^6 \text{ m s}^{-1}$

6. $1.6 \times 10^{-19} \text{ C}$
7. 9.1 T
8. (a) $2.1 \times 10^{-13} \text{ N}$
 (b) This force is a central force at right angles to the direction of motion.
 (c) $2.9 \times 10^{-4} \text{ m}$
9. 0.54 m
10. (a) 45 mm
 (b) $2.9 \times 10^{-16} \text{ N}$
11. (a) $2.6 \times 10^7 \text{ m s}^{-1}$
 (b) $1.1 \times 10^{-7} \text{ s}$
 (c) $2.2 \times 10^{-12} \text{ J}$
12. $1.9 \times 10^6 \text{ m s}^{-1}$
13. 0.61 T
14. $9.56 \times 10^7 \text{ C kg}^{-1}$; proton, q/m for proton = $9.56 \times 10^7 \text{ C kg}^{-1}$
15. (a) $v \cos\theta$
 (b) $v \sin\theta$
 (c) $v \cos\theta$ stays unchanged, as it is parallel to the magnetic field
16. (a) $2.3 \times 10^6 \text{ m s}^{-1}$
 (b) $6.4 \times 10^6 \text{ m s}^{-1}$
 (c) $2.36 \times 10^{-13} \text{ N}$
 (d) $1.6 \times 10^{-4} \text{ m}$
 (e) $1.6 \times 10^{-10} \text{ s}$
 (f) $3.7 \times 10^{-4} \text{ m}$
17. (a) $4.4 \times 10^5 \text{ m s}^{-1}$
 (b) $3.7 \times 10^5 \text{ m s}^{-1}$
 (c) $2.8 \times 10^{-14} \text{ N}$
 (d) $8.2 \times 10^{-3} \text{ m}$
 (e) $1.4 \times 10^{-7} \text{ s}$
 (f) $6.2 \times 10^{-2} \text{ m}$
18. (a) $1.2 \times 10^{-4} \text{ m}$
 (b) $5.4 \times 10^{-4} \text{ m}$

Simple harmonic motion

1. (a) The unbalanced force is proportional to the displacement, and acts in the opposite direction.
- (b) A mass oscillating at the end of a spring, a pendulum with small amplitude, a hack saw blade fixed at one end and vibrating at the other.

2. (a) $\frac{d^2y}{dt^2} = -\omega^2 y$

(b) (i)

$$y = a \cos \omega t \text{ if } y = 0 \text{ at } t = 0 \quad \text{and} \quad y = a \sin \omega t \text{ if } y = a \text{ at } t = 0$$

Acceleration

Differentiating $\frac{dy}{dt} = \frac{d}{dt}(a \cos \omega t)$ $= -a\omega \sin \omega t$ Differentiating again $\frac{d^2y}{dt^2} = -a\omega^2 \cos \omega t$ but $y = a \cos \omega t$ $\frac{d^2y}{dt^2} = -\omega^2 y$		$\frac{dy}{dt} = \frac{d}{dt}(a \sin \omega t)$ $= a\omega \cos \omega t$ $\frac{d^2y}{dt^2} = -a\omega^2 \sin \omega t$ $\frac{d^2y}{dt^2} = -\omega^2 y \quad (y = a \sin \omega t)$
--	--	---

(ii) $y = a$ at $t = 0$ and $y = 0$ at $t = 0$.

(c)

Velocity

$v = \frac{dy}{dt} = -a\omega \sin \omega t$ $v^2 = a^2\omega^2 \sin^2 \omega t$ and $y^2 = a^2 \cos^2 \omega t$ $\sin^2 \omega t + \cos^2 \omega t = 1$ Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$ $v^2 = \omega^2(a^2 - y^2)$ Thus $v = \pm \omega \sqrt{a^2 - y^2}$		$v = \frac{dy}{dt} = a\omega \cos \omega t$ $v^2 = a^2\omega^2 \cos^2 \omega t$ and $y^2 = a^2 \sin^2 \omega t$ $\cos^2 \omega t + \sin^2 \omega t = 1$ Thus $\frac{v^2}{a^2\omega^2} + \frac{y^2}{a^2} = 1$ $v^2 = \omega^2(a^2 - y^2)$ Thus $v = \pm \omega \sqrt{a^2 - y^2}$
--	--	---

3. (a) acceleration = $-\omega^2 y$ acceleration at centre = 0 ($y = 0$)
 acceleration at extremities = $\pm 4\pi^2 \times 5^2 \times 0.04 = \pm 39 \text{ m s}^{-2}$ ($\omega = 2\pi f$)
- (b) velocity = $\pm \omega \sqrt{a^2 - y^2}$
 velocity at extremities = 0
 velocity at centre = $\pm \omega a = \pm 2\pi \times 5 \times 0.04 = \pm 1.3 \text{ m s}^{-1}$
- (c) acceleration = $\pm 4\pi^2 \times 5^2 \times 0.02 = \pm 20 \text{ m s}^{-2}$
 velocity = $\pm 2\pi \times 5 \times \sqrt{0.04^2 - 0.02^2} = \pm 1.1 \text{ m s}^{-1}$

4. Acceleration of oscillations must be numerically less than $g = 9.8 \text{ m s}^{-2}$
 $9.8 = \omega^2 a = 2\pi \times \frac{1}{0.1} \times a$ giving $a = 0.16 \text{ m}$

5 (a)

Kinetic energy equation for the particle

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} m [\pm \omega \sqrt{a^2 - y^2}]^2$$

$$E_k = \frac{1}{2} m \omega^2 (a^2 - y^2)$$

Potential energy equation for the particle

When at position O the potential energy is zero, (with reference to the equilibrium position) and the **kinetic energy is a maximum**.

The kinetic energy is a maximum when $y = 0$: $E_{k\max} = \frac{1}{2} m \omega^2 a^2$

At point O total energy $E = E_k + E_p = \frac{1}{2} m \omega^2 a^2 + 0$

$$E = \frac{1}{2} m \omega^2 a^2 \quad \text{or} \quad E = \frac{1}{2} k a^2 \quad \text{because } \omega^2 = \frac{k}{m}$$

The **total energy E** is the **same at all points** in the motion.

Thus for any point on the swing: as above $E = E_k + E_p$

$$\frac{1}{2} m \omega^2 a^2 = \frac{1}{2} m \omega^2 (a^2 - y^2) + E_p$$

$$E_p = \frac{1}{2} m \omega^2 y^2$$

- (b) $E_k = \frac{1}{2} m\omega^2(a^2 - y^2)$ $E_p = \frac{1}{2} m\omega^2 y^2$
- (i) $E_k(\text{max}) = \frac{1}{2} \times 0.2 \times (2\pi \times 0.5)^2 \times 0.1^2 = 9.9 \times 10^{-3} \text{ J}$ at the central position
- (ii) $E_k(\text{min}) = 0$ when $y = a$, at the extreme position
- (iii) $E_p(\text{max}) = \frac{1}{2} \times 0.2 \times (2\pi \times 0.5)^2 \times 0.1^2 = 9.9 \times 10^{-3} \text{ J}$ at the extreme position
- (iv) $E_k(\text{min}) = 0$ at the central position when $y = 0$
- (v) Sum of E_p and E_k at any point is always the same and equal to $9.9 \times 10^{-3} \text{ J}$

6. (a) 0.44 mm

(b) $28 = 2\pi f$ $f = 4.5 \text{ Hz}$

(c) period $T = 1/f = 0.22 \text{ s}$

(d) $0.2 = 0.44 \sin 28t$ hence $\sin 28t = \frac{0.2}{0.44} = 0.4545$

$28t = 0.472$ (remember to work in radians) giving $t = 0.017 \text{ s}$

7. (a) Reduces the amplitude of the oscillations

(b) When a system is critically damped the frictional resistance is just enough to prevent any oscillations occurring. The time taken for the displacement to become zero is a minimum.

When a system is overdamped the frictional resistance is large and the system takes a long time to come to rest.

(c) Critical damping for the needle on a meter.

Critical damping for car shock absorbers

Simple harmonic motion

1 (a) $y = 4 \cos 4\pi t$ compare with $y = a \cos \omega t$
 thus $a = y_{\max} = \underline{4}$ cm

(b) $\omega = 4\pi$ and $\omega = 2\pi f$
 $4\pi = 2\pi f$ giving $f = \underline{2}$ Hz

Also $f = \frac{1}{T}$ thus $T = \underline{0.5}$ s

(c) (i) when $t = 0$ s $y = 4 \cos 0 = \underline{4}$ cm

(ii) when $t = 1.5$ s $y = 4 \cos (4\pi \times 1.5)$ (Remember angle in radians)
 $= \underline{4}$ cm

2 (a) $f = 40$ Hz thus $T = \frac{1}{f} = \frac{1}{40} = \underline{0.025}$ s

(b) $y = a \cos \omega t = a \cos 2\pi ft = \underline{0.05 \cos 80\pi t}$

(c) (i) acceleration = $-\omega^2 y$

at mid-point $y = 0$ thus acceleration = $\underline{0}$ m s⁻²

at max amplitude $y = 0.05$ m

acceleration = $-(2\pi \times 40)^2 \times 0.05$

$= -\underline{3.2 \times 10^3}$ m s⁻² directed towards the midpoint

(ii) $v_{\max} = \pm \omega a = \pm 2\pi \times 40 \times 0.05$

$= \underline{\pm 12.6}$ m s⁻¹ this occurs at the midpoint when $y = 0$.

3 (a) $y = a \cos \omega t$ (at $t = 0$, $y = a$)
 $= 0.12 \cos \frac{2\pi}{1.5} \times 0.4$ (remember ωt in radians)
 $= \underline{-0.0125}$ m (or $y = -0.013$ m to 2 sig figs)

The position of object is 0.0125 m on the **opposite** side of the equilibrium position to $y = a$ at $t = 0$.

(b) Use $F = m \times \text{acceleration}$ and $\text{acceleration} = -\omega^2 y$
 $= m \times -\omega^2 y$
 $= -0.5 \times \left(\frac{2\pi}{1.5}\right)^2 \times 0.0125$
 $= \underline{0.11}$ N

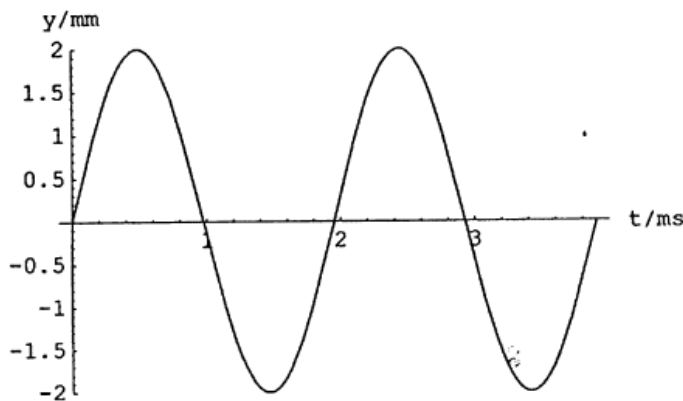
The force is acting in the positive direction, towards the equilibrium position.

(c) $y = -0.06$ m and $-0.06 = 0.12 \times \cos\left(\frac{2\pi}{1.5} \times t\right)$
 $\cos\left(\frac{2\pi}{1.5} \times t\right) = -\frac{0.06}{0.12}$ and $\frac{2\pi}{1.5} \times t = \cos^{-1}\left(-\frac{0.06}{0.12}\right)$
 $t = \frac{1.5}{2\pi} \cos^{-1}\left(-\frac{0.06}{0.12}\right)$
 $t = \underline{0.5}$ s

4 (a) $y = a \sin \omega t$ and $y_{\max} = a$
 $y = 2.0 \sin (3.22 \times 10^3 t)$ thus $y_{\max} = \underline{2.0}$ mm
 $\omega = 2\pi f$ and $\omega = 3.22 \times 10^3$
 $f = \frac{3.22 \times 10^3}{2\pi} = \underline{512}$ Hz

(b) $\text{accn}_{\max} = -\omega^2 y_{\max} = -(2\pi \times 512)^2 \times 2.0 \times 10^{-3}$
 $= \underline{2.07 \times 10^4}$ m s⁻²

(c) $T = \frac{1}{512} = 1.95 \times 10^{-3}$ s and $y = 0$ when $t = 0$



(d) The period of any SHM is constant even although the amplitude is decreasing.

5 Acceleration will have to be greater than 10 m s^{-2} for this condition to occur.

Use $\text{accn}_{\max} = -\omega^2 y_{\max}$ and $\omega = 2\pi f$
 $= (2 \times \pi \times 40)^2 y_{\max}$
 $y_{\max} = \frac{10}{(2 \times \pi \times 40)^2} = \underline{1.58 \times 10^{-4}}$ m

6 (a) $k = \frac{\text{force}}{\text{extension}} = \frac{1.2 \times 9.8}{0.10} = \underline{118}$ N m⁻¹

(b) (i) amplitude = 0.08 m

(ii) $\omega^2 = \frac{k}{m} = \frac{118}{1.2}$ and $T = \frac{2\pi}{\omega}$

$T = 2\pi \sqrt{\frac{1.2}{118}} = \underline{0.63}$ s and $f = \frac{1}{T} = \underline{1.6}$ Hz

(iii) $v_{\max} = \pm \omega a = \pm \sqrt{\frac{118}{1.2}} \times 0.08 = \underline{\pm 0.79}$ m s⁻¹

Total energy = $\frac{1}{2} m \omega^2 a^2 = \frac{1}{2} \times 1.2 \times \frac{118}{1.2} \times 0.08^2$
 $= \underline{0.38}$ J

- 7 First use the conservation of linear momentum to find the velocity just after the dart embeds.

In the absence of external forces;

total momentum before = total momentum after

$$mv_1 = (m + M) \times v_2$$

$$0.060 \times 120 = (5.0 + 0.06) \times v_2$$

$$v_2 = \frac{0.060 \times 120}{5.06} = 1.42 \text{ m s}^{-1}$$

This is the maximum velocity, v_{\max}

$$(a) \quad v_{\max} = \omega a \quad \text{and} \quad \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{450}{5.06}}$$

$$a = \frac{v_{\max}}{\omega} = 1.42 \sqrt{\frac{5.06}{450}} = \underline{0.15 \text{ m}}$$

An alternative solution can be found from an analysis of the energy.

$$E_{\text{tot}}(\text{system}) = \frac{1}{2} m v_{\max}^2 = \frac{1}{2} \times 5.06 \times (1.42)^2$$

$$= 5.10 \text{ J}$$

but $E_p = \frac{1}{2} k a^2$ when $v = 0$ i.e. $E_k \rightarrow E_p$

$$\text{thus} \quad a^2 = \frac{2 \times E_{\text{tot}}}{k} \quad \text{and} \quad a = \sqrt{\frac{2 \times 5.10}{450}} = \underline{0.15 \text{ m}}$$

(b) $E_k(\text{total of system}) = 5.1 \text{ J}$ (see above $E_{\text{tot}} = \frac{1}{2} m v_{\max}^2$)

$$E_k(\text{of dart}) = \frac{1}{2} m v^2 = \frac{1}{2} \times 0.06 \times (120)^2$$

$$= 432 \text{ J}$$

$$\text{percentage of energy in oscillating system} = \frac{5.1}{432} \times \frac{100}{1} = \underline{1.2 \%}$$

- 8 An oscillating system will eventually come to rest if there is no driving force. The amplitude of the oscillations decrease due to the presence of friction. This decrease in the amplitude is called damping.

Critical damping occurs when the frictional resistance is just sufficient to prevent any oscillations past the rest position. Critical damping occurs when an oscillating system comes to rest in the shortest possible time.

It is worth noting that in the process of damping the energy of the system ends up as heat energy which is transferred to the surroundings.

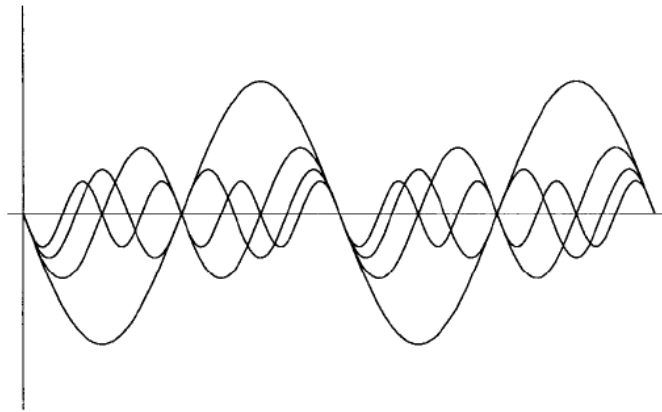
Waves

- (a) intensity \propto (amplitude)²
(b) an increase of 81 times
- Any waveform can be represented by a series of sine or cosine expressions.

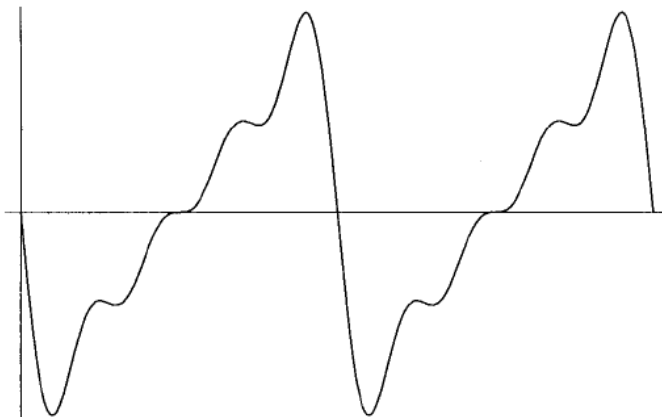
A **periodic** wave is a wave which repeats itself at regular intervals. All periodic waveforms can be described by a mathematical series of sine or cosine waves, known as a Fourier Series. For example a saw tooth wave can be expressed as a series of individual sine waves.

$$y(t) = -\frac{1}{\pi} \sin \omega t - \frac{1}{2\pi} \sin 2\omega t - \frac{1}{3\pi} \sin 3\omega t - \dots$$

The graph below shows the first four terms of this expression.



When all these terms are superimposed (added together) the graph below is obtained. Notice that this is tending to the sawtooth waveform. If more terms are included it will have a better saw tooth form.



3. (a) y – displacement in transverse direction
 a – amplitude f – frequency t – time
 x – distance of a particle from the origin λ - wavelength
- (b) (i) 0.06 m
(ii) 75 Hz [using $2\pi f = \pi \times 150$]
(iii) 13 ms [using $T = 1/f$]
(iv) 5 m [using $2\pi/\lambda = \pi \times 0.40$]
(v) 375 m s^{-1} [using $v = \lambda f$]
4. (a) amplitude, frequency, period
(b) no, at time $t = 0$ the displacements are different.
5. (a) A stationary wave does not travel to the left or the right, but particle displacements do still take place. The particle displacements increase or decrease in unison. In some places there are maximum amplitudes in other places zero amplitude and no vibration.
(b) A node is a position of zero amplitude.
An antinode is a position of maximum amplitude.

Waves

1 (a) $y = 3 \sin 2\pi (10t - 0.2x)$ is compared with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$$a = \underline{3.0} \text{ cm}$$

(b) $f = \underline{10} \text{ Hz}$

(c) $\frac{1}{\lambda} = 0.2$ thus $\lambda = \frac{1}{0.2} = \underline{5.0} \text{ cm}$

(d) $v = f\lambda = 10 \times 5.0 = \underline{50} \text{ cm s}^{-1}$

2 $y = a \sin 2\pi (ft - \frac{x}{\lambda})$ thus $y = 0.30 \sin 2\pi (20t - \frac{x}{0.5})$

3 $y = 0.20 \sin (220\pi t - 30\pi x)$ rewrite in the form $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$$y = 0.20 \sin 2\pi (110t - 15x)$$

assuming v remains the same, doubling f halves λ

thus $y = 0.40 \sin 2\pi (220t + 30x)$

4 $y = 0.04 \sin [2\pi(\frac{t}{0.04} - \frac{x}{2.0})]$

(a) $y_{\text{max}} = \underline{0.04} \text{ m}$

(b) compare with $y = a \sin 2\pi (ft - \frac{x}{\lambda})$

$$\lambda = \underline{2.0} \text{ m}$$

(c) $f = \frac{1}{0.04} = \underline{25} \text{ Hz}$

(d) The movement of a particle will be Simple Harmonic with a maximum amplitude of 0.04 m. The particle will move in a direction perpendicular to the wave direction along the string.

5 (a) $y = 0.01 \sin \pi (2.0t - 0.01x)$

$$= 0.01 \sin 2\pi (t - \frac{0.01}{2} x)$$

thus $f = 1.0 \text{ Hz}$ and $\lambda = \frac{2}{0.01} = 200 \text{ m}$

$$v = f\lambda = 1.0 \times 200 = \underline{200} \text{ m s}^{-1}$$

(b) at $x = 0$ $y = 0.01 \sin \pi (2.0t)$

$$= 0.01 \sin 2\pi t$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt} (0.01 \sin 2\pi t) = [0.01 \cos 2\pi t] 2\pi$$

thus $v_{y\text{max}} = 0.01 \times 1.0 \times 2\pi = \underline{0.063} \text{ m s}^{-1}$

6 (a) $y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$

substitute $f = \frac{1}{T}$ giving $y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right)$

(b) substitute $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$

$$y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right) = y = a \sin \left(2\pi ft - \frac{2\pi x}{\lambda} \right)$$

$$y = a \sin (\omega t - kx)$$

(c) substitute $\lambda = \frac{v}{f}$ into $y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$

$$y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right) = y = a \sin 2\pi \left(ft - \frac{xf}{v} \right)$$

$$y = a \sin 2\pi f \left(t - \frac{x}{v} \right)$$

(d) substitute $f = \frac{v}{\lambda}$ into $y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$

$$y = a \sin 2\pi \left(\frac{vt}{\lambda} - \frac{x}{\lambda} \right)$$

$$y = a \sin \frac{2\pi}{\lambda} (vt - x)$$

7 (a) A phase difference of 2π occurs in one wavelength $\lambda = \frac{v}{f} = \frac{350}{500} = 0.70 \text{ m}$

thus a phase difference of $\frac{\pi}{3}$ occurs in $\frac{1}{6} \lambda$

the two points are separated by $\frac{0.70}{6} = \underline{0.12} \text{ m}$

(Alternatively use $\phi = \frac{2\pi x}{\lambda}$)

(b) 2π phase difference occurs in one period (T)

$$T = \frac{1}{f} = \frac{1}{500} = 0.002 \text{ s}$$

0.002 s is equivalent to a phase difference of 2π

0.001 s is equivalent to a phase difference of π

8 (a) $\lambda = \frac{v}{f} = \frac{30}{250} = 0.12 \text{ m}$

$$\text{phase difference} = \frac{2\pi x}{\lambda} = 2\pi \times \frac{10}{12} = \underline{1.67\pi} \text{ radians} = \underline{5.24} \text{ radians}$$

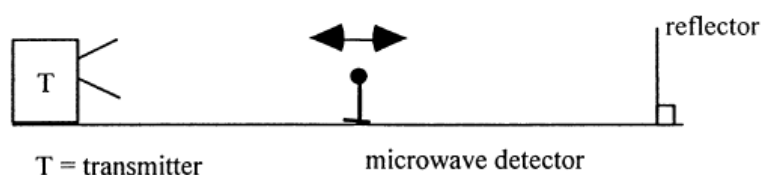
(b) $y = a \sin 2\pi \left(ft - \frac{x}{\lambda} \right) = 0.03 \sin 2\pi \left(250t - \frac{x}{0.12} \right)$

(c) distance between nodes $= \frac{\lambda}{2} = \frac{0.12}{2} = \underline{0.06} \text{ m}$

- 9 (a) A travelling wave is a wave which moves through a material transferring energy in the direction of travel. All particles of the material which transmits the energy perform Simple Harmonic Motion.

A stationary wave has parts of the material at rest, the nodes, and energy does not travel along the material. Energy is effectively trapped between the nodes. The particles between the nodes vibrate in phase with SHM but have different amplitudes.

- (b) Wavelength of microwaves



Set up the apparatus as above. Move the microwave detector between transmitter and reflector and note that nodes are detected.

Measure the distance, d , between the first and eleventh node for example.

The distance between adjacent nodes = $\frac{\lambda}{2}$

Thus distance between first and eleventh nodes is 5λ giving $\lambda = \frac{d}{5}$.

- 10 (a) (i) The microphone has been moved a distance of $\frac{\lambda}{2}$.

thus $\lambda = 2 \times 0.24 = \underline{0.48} \text{ m}$

(ii) $v = f\lambda = 700 \times 0.48 = 336 \text{ m s}^{-1}$

velocity of sound = 340 m s^{-1} (2 sig figs)

- (b) As the listener walks across the room he will hear alternately quiet and loud sounds of the same frequency. This is because there are two sources of sound producing coherent waves and as they overlap, constructive and destructive interference takes place. Constructive interference will be positions of loud sound and destructive interference positions of quiet.

At constructive interference the path difference is $n\lambda$, the waves are in phase, the amplitude is bigger so there is more energy. At destructive interference the path difference is $(n + \frac{1}{2})\lambda$, the waves are completely out of phase, the amplitude is zero so there is no energy. The quiet patches are not completely silent because there is a degree of reflection of the sound from the walls in the room.

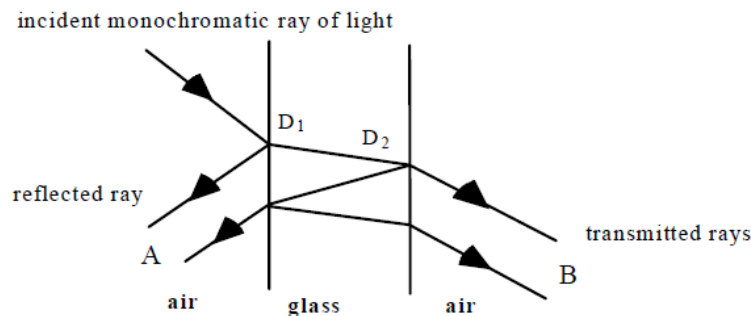
Interference – division of amplitude

1. (a) Constant phase difference.
 (b) Light is produced when electrons, which have been excited, return to a lower energy state. This is a random process in that two separate sources will not emit light beams which have a constant phase difference, even if they have the same frequency.
 (c) yes, both loudspeakers are driven by the same single source. Any change in phase from the single source occurs simultaneously at the loudspeakers.

2. (a) Optical path difference = geometric path difference x refractive index
 (b) phase difference = $\frac{2\pi}{\lambda}$ x optical path difference
 (c) (i) optical path AB = $(80 \times 1.5) + (150 - 80) = 190$ nm
 (ii) optical path difference = $190 - 150 = 40$ nm
 phase difference = $\frac{2\pi}{700 \times 10^{-9}} \times 40 \times 10^{-9} = 0.36$ radians

3. (a) π radians (or 180°)
 (b) None, the water has a smaller refractive index than the glass.

4. (a) Light incident on the film. The amplitude of the ray is divided. The light is partially reflected and partially refracted at D_1 . The reflected rays at A have a different path difference so will interfere when brought together, similarly for the transmitted rays at B.

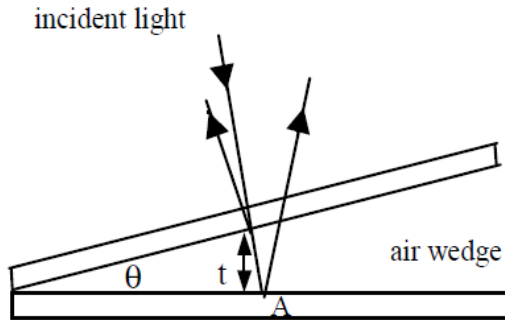


- (b) (i) $2nt = m\lambda$ for destructive interference
 (ii) When t increases the value of λ needed to produce constructive interference will increase. The colour of the pattern will move towards the red end of the spectrum.

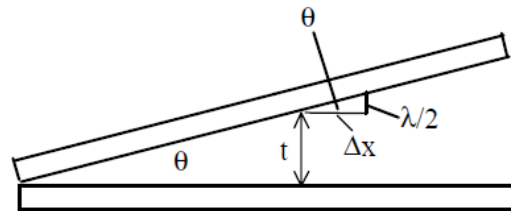
5 (a)

Two glass slides are arranged as shown below.

Division of amplitude takes place at the *lower* surface of the top glass slide.



Enlarged view showing the geometry



When viewed from above the optical path difference = $2t$

There is a phase difference of π on reflection at A. Hence the condition for a dark fringe is $2t = m\lambda$ assuming an air wedge.

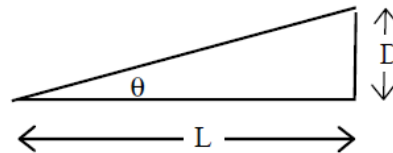
For the next dark fringe t increases by $\frac{\lambda}{2}$ (see right hand sketch above).

Thus the spacing of fringes, Δx , is such that $\tan\theta = \frac{\lambda}{2\Delta x}$ giving

$$\Delta x = \frac{\lambda}{2 \tan\theta}$$

For a wedge of length L and spacing D

$$\tan\theta = \frac{D}{L}$$



The fringe spacing is given by

$$\Delta x = \frac{\lambda L}{2D}$$

where λ is the wavelength of light in air.

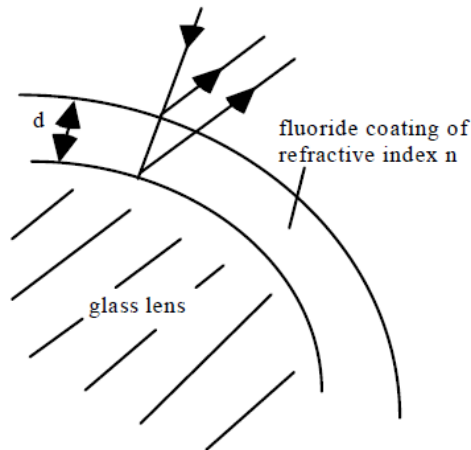
(b) $\Delta x = \frac{\lambda L}{2D} = \frac{650 \times 10^{-9} \times 100 \times 10^{-3}}{2 \times 30 \times 10^{-6}} = 1.1 \text{ mm}$

(c) The thickness of the paper has increased. The wavelength and length of the plates are constant.

6 (a)

Good quality lenses in a camera reflect very little light and appear dark or slightly purple. A thin coating of a fluoride salt such as magnesium fluoride on the surface of the lens allows the majority of the light falling on the lens to pass through.

The refractive index, n , of the coating is chosen such that $1 < n < n_{\text{glass}}$.



Notice that there is a phase change of π at both the first and second surfaces.

For cancellation of reflected light:

$$\text{optical path difference} = \frac{\lambda}{2}$$

$$\text{Optical path in fluoride} = 2nd$$

$$\text{thus } 2nd = \frac{\lambda}{2} \text{ and}$$

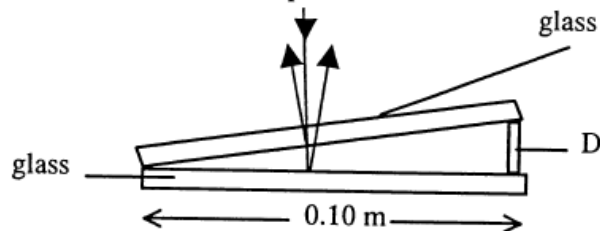
$$d = \frac{\lambda}{4n}$$

(b) $d = \frac{\lambda}{4n} = 8.8 \times 10^{-8} \text{ m}$

(c) The non-reflective coating will only give complete cancellation for one particular wavelength. For a coating giving cancellation for green light, the blue and red would be partially reflected and the lens would appear purple.

Interference - division of amplitude

- 1 (a) Coherent sources must have a constant phase relationship. The two or more sources will come from the same original source.
 - (b) When we try to produce an interference pattern from two separate light sources it does not work because light is produced in small wave packets and not as a continuous wave. This is not the case for sound waves. We can have two separate loudspeakers, connected to the same signal generator, and produce an interference pattern.
- 2 (a) Division of amplitude involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at a surface between two materials of different refractive index. They may be multiple reflections and transmissions.
 - (b) An extended beam of light can be used because the beam is sub-divided by reflection and transmission at a surface. Hence there will always be a fixed phase relation between the sub-divided parts.
- 3 (a) Fringes are formed when reflections from the bottom surface of the top glass plate interfere with the beam transmitted through the top plate and reflected from the top surface of the bottom plate, see sketch below.



(b) $\tan \theta = \frac{\lambda}{2\Delta x}$ where $\lambda =$ wavelength and $\Delta x =$ fringe spacing

$$= \frac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}}$$

from sketch $D = \tan \theta \times 0.10 = \frac{6.9 \times 10^{-7}}{2 \times 1.2 \times 10^{-3}} \times 0.10$ $D =$ thickness of foil

$$= \underline{2.9 \times 10^{-5} \text{ m}}$$

(c) new value of $D = 2.9 \times 10^{-5} \times 1.1 = 3.2 \times 10^{-5} \text{ m}$ i.e. 10% bigger

new $\tan \theta = \frac{3.2 \times 10^{-5}}{0.10}$

new $\Delta x = \frac{\lambda}{2 \tan \theta} = \frac{6.9 \times 10^{-7} \times 0.10}{2 \times 3.2 \times 10^{-5}}$

$$= \underline{1.1 \times 10^{-3} \text{ m}}$$

4 (a) For cancellation of reflected light: optical path in fluoride = $\frac{\lambda}{2}$

thus $2nd = \frac{\lambda}{2}$ where n = refractive index and d is the thickness of the film

$$d = \frac{\lambda}{4n}$$

(b) $d = \frac{\lambda}{4n} = \frac{4.8 \times 10^{-7}}{4 \times 1.25} = \underline{9.6 \times 10^{-8}} \text{ m}$

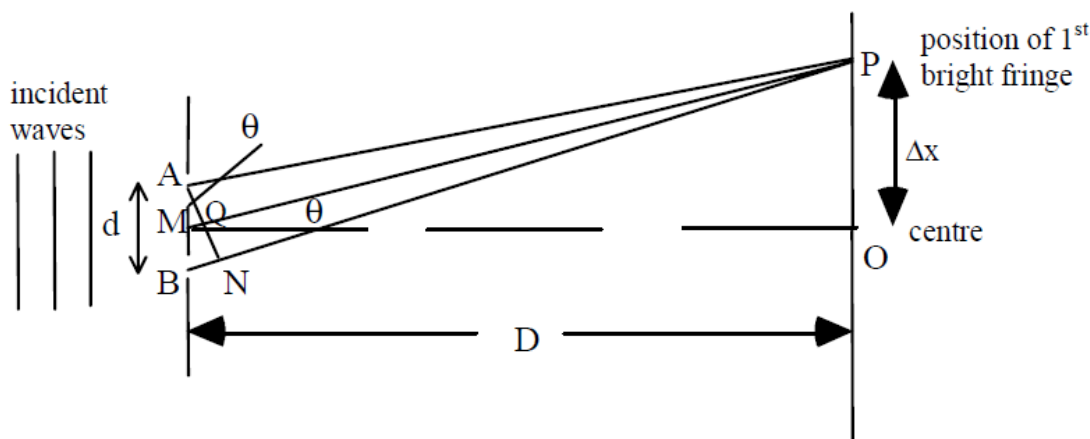
5 $d = \frac{\lambda}{4n} = \frac{6.7 \times 10^{-7}}{4 \times 1.3} = \underline{1.3 \times 10^{-7}} \text{ m}$

6 $d = \frac{\lambda}{4n} = \frac{6.2 \times 10^{-7}}{4 \times 1.3} = \underline{1.2 \times 10^{-7}} \text{ m}$

Interference – division of wavelength

1. (a) When light is incident on two small slits, the wavefront is divided and each slit acts as a secondary source. Interference takes place between the two secondary sources.
 - (b) With an extended source each part of the wave would be incident on the slit at a different angle which could produce overlapping fringes and the interference pattern would be lost. A point source (or a line source parallel to the slits) must be used.
 - (c) With division of amplitude the beam is split at a point with partial reflection and transmission.
- 2 (a)

The diagram below shows light from a single source of monochromatic light incident on a double slit. The light diffracts at each slit and the overlapping diffraction patterns produce interference.



A bright fringe is observed at P. Angle PMO is θ .

N is a point on BP such that $NP = AP$. Since P is the first bright fringe $BN = \lambda$

For **small** values of θ AN cuts MP at almost 90° giving angle $MAQ = \theta$ and hence angle $BAN = \theta$.

Again providing θ is **very small**, $\sin \theta = \tan \theta = \theta$ in radians

From triangle BAN: $\theta = \frac{\lambda}{d}$ also from triangle PMO: $\theta = \frac{\Delta x}{D}$

$$\text{Thus } \frac{\Delta x}{D} = \frac{\lambda}{d} \text{ or } \Delta x = \frac{\lambda D}{d}$$

Giving the fringe separation between adjacent fringes Δx

$$\Delta x = \frac{\lambda D}{d}$$

Note This formula only applies if $x \ll D$, which gives θ small. This is likely to be true for light waves but not for microwaves.

(b) $x \ll D$ giving θ a small angle and $\sin\theta = \theta = \tan\theta$

3. (a) $\Delta x = \frac{\lambda D}{d}$ giving $\lambda = 6.0 \times 10^{-7} \text{ m}$

(b) % uncertainty in: Δx is 1.1%; d is 4.2%; D is 2.6%

The 1.1% can be neglected since less than one third of 4.2%

$$\text{Total uncertainty} = \sqrt{4.2^2 + 2.6^2} = 4.9\%$$

$$\text{Uncertainty in the wavelength} = 0.3 \times 10^{-7} \text{ m}$$

$$\lambda = (6.0 \pm 0.3) \times 10^{-7} \text{ m}$$

4. (a) Increase the slide to screen distance D .

(b) Fringes are further apart.

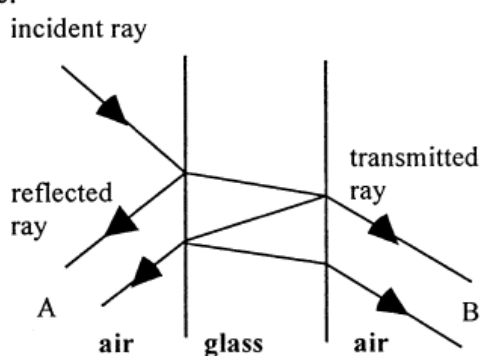
(c) The fringes are further apart.

(d) A travelling microscope

(e) Measure the distance across a number of fringes, for example ten, then calculate the fringe spacing.

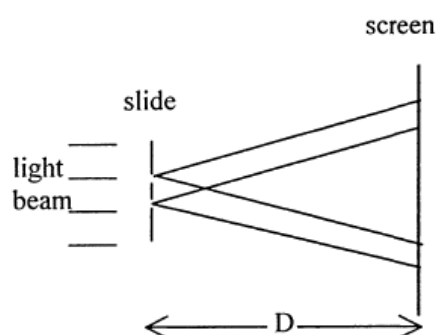
Interference – division of wavefront

- 1 An example of **division of amplitude** involves splitting a single beam into two beams by producing a reflected beam and a transmitted beam at thin parallel sided film. The reflected rays at A will give interference. Similarly the transmitted rays at B will give interference. In both cases the rays can be brought to a focus with the eye.



An extended source of light can be used. Any transparent boundary between two media of different refractive index can produce division of amplitude.

An example of **division of wavefront** involves a single source incident on a double slit producing two secondary sources. The two secondary beams will give interference fringes on a screen, see sketch below which is not to scale. The distance between the slits must be very small and the distance D is a few metres.



The source of light must be a point or line source. The slits size must be the order of the wavelength to act as secondary sources.

- 2 (a) The two narrow slits act as coherent sources by division of wavefront. When light from each of the sources meet in phase areas of constructive interference are produced, giving a bright fringe. When the light from the two sources are completely out of phase there is destructive interference and almost darkness.
- (b) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the n th interference fringe is given by: $x_n = \frac{n\lambda D}{d}$

Notice that the position of a fringe is dependent on the wavelength, λ ; as λ increases x_n increases. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

3 (a)
$$\Delta x = \frac{\lambda D}{d} \quad \Delta x = \frac{695 \times 10^{-9} \times 0.92}{2.0 \times 10^{-4}}$$

$$\Delta x = 3.2 \times 10^{-3} \text{ m} = \underline{3.2} \text{ mm}$$

- (b) The new double slit is half the size. From the relationship in (a) above the fringe separation and the slit separation are inversely proportional. This means that decreasing d will increase x . The pattern will spread out and the fringes will be further apart. In this case the spacing of the fringes will double to 6.4 mm.

4

$$\Delta x = \frac{\lambda D}{d} \quad \text{thus} \quad \lambda = \frac{\Delta x d}{D}$$

$$\lambda = \frac{8 \times 10^{-3} \times 5.0 \times 10^{-4}}{7.2}$$

$$= 5.56 \times 10^{-7} = \underline{556 \text{ nm}}$$

- 5 (a) Two coherent sources are produced by the double slit. Interference takes place between these two sources and red and black lines called fringes will be seen on the screen. With a red filter only red fringes are seen because all of the other wavelengths, except red, are absorbed by the filter.
 A bright red line is formed by constructive interference. The path difference is $n\lambda$, the waves arrive in phase, there is a larger amplitude and more energy.
 A dark line is formed by destructive interference. The path difference is $(n + \frac{1}{2})\lambda$, the waves are completely out of phase, the amplitude is zero and there is no energy.

- (b) The blue fringes will be closer together because the fringe separation Δx is proportional to the wavelength λ and $\lambda_{\text{blue}} < \lambda_{\text{red}}$.

- (c) White fringes have coloured edges because white light is composed of the colours of the spectrum. The position of the n th interference fringe is given by: $x_n = \frac{n\lambda D}{d}$
 Notice that this is dependent on λ the wavelength. Thus red will be deviated most and violet least. These two colours will appear at the edges of the white fringes.

(d)

$$\lambda = \frac{\Delta x d}{D}$$

$$= \frac{5 \times 10^{-3} \times 0.25 \times 10^{-3}}{2.0}$$

$$= 6.25 \times 10^{-7} = \underline{625 \text{ nm}}$$

6 (a) (i)
$$\Delta x = \frac{\lambda D}{d}$$

Δx is proportional to D . Thus making D smaller will also reduce Δx .

- (ii) Δx is inversely proportional to d . Thus decreasing d will increase Δx , making the fringes further apart.

- (b) (i) Covering one of the slits will cause the interference patterns to disappear. Two sources are needed to produce interference in division of wavefront.

(ii)
$$\Delta x \propto \lambda$$

Thus a longer wavelength will produce a larger Δx and the fringes will be further apart.

- (iii) White light fringes will be seen. Depending on the values of λ , D and d the fringes may have red and violet coloured edges, see answer to 5 (c).

$$\begin{aligned}
 \text{(c)} \quad \lambda &= \frac{\Delta x d}{D} \\
 &= \frac{10 \times 10^{-3} \times 0.5 \times 10^{-3}}{8.0} \\
 &= 6.25 \times 10^{-7} = \underline{625} \text{ nm}
 \end{aligned}$$

- (d) The fringe separation would have the greatest percentage uncertainty because this is likely to be measured using a metre stick giving (10 ± 1) mm which is 10%. The slit separation can be measured to a higher accuracy with a travelling microscope. The distance to the screen is much larger than the fringe separation so the percentage uncertainty for this will be much less, namely $8 \text{ m} \pm 1 \text{ mm}$ which is 0.01%

$$\begin{aligned}
 7 \text{ (a) (i)} \quad \lambda &= \frac{\Delta x d}{D} = \frac{7 \times 10^{-3} \times 0.2 \times 10^{-3}}{2.4} \\
 &= 5.8 \times 10^{-7} = \underline{580} \text{ nm}
 \end{aligned}$$

$$\text{(ii) uncertainty in } \Delta x = \frac{1}{7} \times 100 = 14 \%$$

$$\text{uncertainty in } d = \frac{0.01}{0.20} \times 100 = 5 \%$$

$$\text{uncertainty in } D = \frac{0.10}{2.4} \times 100 = 4 \%$$

The uncertainty in D can be neglected, since it is less than $\frac{1}{3}$ of 14 %.

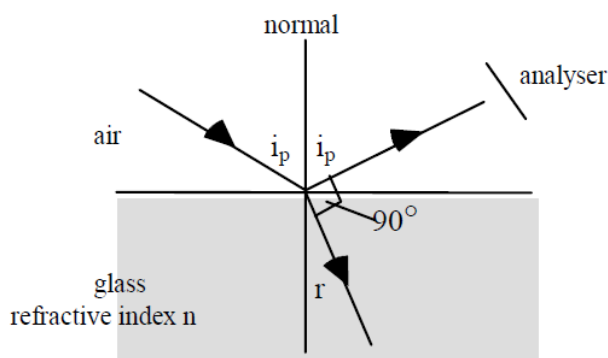
$$\text{Total \% uncertainty} = \sqrt{14^2 + 5^2} = 14.9 \%$$

Thus uncertainty in the wavelength is 86 nm and $\lambda = (\underline{5.8} \pm \underline{0.9}) \times 10^{-7} \text{ m}$

- (b) (i) Place the double slit slide on the stage of a travelling microscope. Focus the cross hairs in the objective of the microscope on the edge of one of the slits ruled on the slide. Sometimes illumination can be a problem, so try putting a low voltage bulb beneath the microscope stage below the position of the slide. The edge of the slit rulings may now be visible. Read this position on the vernier scale. Now rack the microscope along the frame until the crosshairs are at the point which gives the slit separation. Read the new position on the vernier scale. To avoid backlash in the mechanism, do not rack the microscope back and forth; i.e. only move it in one direction when taking both readings.
- (ii) This could be improved by counting a number of fringes and dividing by the number rather than simply measuring the separation of adjacent fringes.
- (c) (i) Reducing d increases the fringe separation Δx because $\Delta x \propto \frac{1}{d}$.
- (ii) Blue light has a shorter wavelength than yellow light. The blue fringes will therefore be closer together than the yellow fringes because $\Delta x \propto \lambda$.
- (iii) Covering one of the slits will cause the interference patterns to disappear. Two coherent sources are needed to produce interference.

Polarisation

1. (a) With linearly polarised light the oscillations of the electric field strength vector are restricted to one plane. With unpolarised light the electric field strength vector oscillates in all directions perpendicular to the direction of wave propagation.
 - (b) A polaroid filter will only transmit vibrations of the electric field vector in one plane.
 - (c) A polariser and analyser are placed at right angle to each other. They are both placed perpendicular to the direction of transmission. The polariser will only transmit vibrations of the electric field vector in one plane, the analyser will absorb these vibrations since they are all perpendicular to its axis of transmission.
2. (a) The medium is an electric insulator.
 - (b)



Consider a beam of unpolarised light incident on a sheet of smooth glass. This beam is partially reflected and partially refracted. The angle of incidence is varied and the reflected ray viewed through an analyser, as shown above. It is observed that at a certain angle of incidence i_p the reflected ray is plane polarised. No light emerges from the analyser at this angle.

The **polarising angle** i_p or **Brewster's angle** is the angle of incidence which causes the reflected light to be linearly polarised.

This effect was first noted by an experimenter called Malus in the early part of the nineteenth century. Later Brewster discovered that **at the polarising angle i_p the refracted and reflected rays are separated by 90° .**

Consider the diagram above, which has this 90° angle marked:

$$n = \frac{\sin i_p}{\sin r}$$

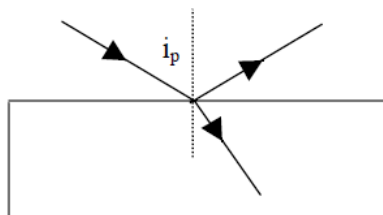
$$\text{but } r = (90 - i_p) \text{ thus } \sin r = \sin (90 - i_p) = \cos i_p$$

$$\text{Thus } n = \frac{\sin i_p}{\cos i_p} = \tan i_p$$

$$\boxed{n = \tan i_p}$$

(c) Brewster's angle

3. (a)



(b) $n = \tan i_p$ $i_p = 56^\circ$

4. When light is reflected from a horizontal surface, such as water, the light will be polarised. The polaroid in the sunglasses acts as an analyser and cuts out a large part of the reflected light. (Note that the light is only completely polarised at the Brewster's angle.)

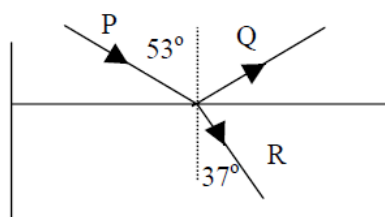
5. (a) $n = \tan i_p$ $i_p = 55^\circ$
 (b) angle of refraction = $90 - 55 = 35^\circ$

6. $n = 1/\sin\theta_c = 1.54$ $n = \tan i_p$ $i_p = 57^\circ$

7. When the frequency increases the refractive index increases. Hence the polarising angle will increase slightly.

8. (a) The intensity of the light observed through the polaroid decreases to a minimum, at the polarising angle, then increases again.

(b) $n = \tan i_p$ $i_p = 53^\circ$
 (c) Angle between ray Q and R is 90°



Polarisation

1 (a) $n = \tan i_p$
 thus $\tan i_p = 1.52$
 $i_p = \underline{57^\circ}$

$r = 90 - i_p$
 $= \underline{33^\circ}$

(b) Hold a polaroid filter, an analyser, at the appropriate angle of 57° and rotate it. At some point in the rotation no light will be transmitted through the analyser.

2 (a) $r = 90 - i_p$
 $= \underline{38^\circ}$

(b) $n = \frac{\sin i}{\sin r} = \frac{\sin 52^\circ}{\sin 38^\circ}$
 $= \underline{1.28}$

3 (a) $n = \tan i_p$
 red - 650 nm $i_p = \tan^{-1} 1.52$
 $= 56.7^\circ$

 green - 510 nm $i_p = \tan^{-1} 1.53$
 $= 56.8^\circ$

 violet - 400 nm $i_p = \tan^{-1} 1.54$
 $= 57^\circ$

(b) $r = 90^\circ - i_p$
 maximum r: $= 90^\circ - 56.7^\circ$
 $= 33.3^\circ$

4 (a) $f = \frac{v}{\lambda} = \frac{3 \times 10^8}{0.028} = \underline{1.1 \times 10^{10}} \text{ Hz}$

(b) The microwave emitter gives out plane polarised waves. The grid in this arrangement acts as an analyser. The metal grid only transmits when the oscillations of the electric field strength vector are perpendicular to the metal rods. Since the initial reading is low, this suggests that the emitted microwaves are plane polarised with their electric vector in the vertical plane.

As the grid is rotated to the horizontal plane, the reading on the receiver will increase because the microwaves will be transmitted. When the grid is rotated further so that it is upside down, to its initial direction, the reading will be a minimum again. There will be a further maximum when the grid is again horizontal.

(c) The venetian blind is acting like the grid in (a) above. Some of the TV waves are being absorbed by the metal slats of the blind.

$$5 \quad n = \frac{1}{\sin c} = \frac{1}{\sin 38^\circ} = 1.62$$

$$i_p = \tan^{-1} 1.62 = \underline{58^\circ}$$

6. (a) When light is incident at a boundary between air and an electrical insulator, the polarising angle i_p is the angle of incidence in air which causes the reflected light to be linearly polarised.

(b) Brewster's angle

(c) Assumption: at the polarising angle the refracted and reflected rays are separated by 90°

$$n = \frac{\sin i_p}{\sin r} \quad \text{and } r = 90 - i_p$$

$$n = \frac{\sin i_p}{\cos i_p} \quad \text{since } \sin(90 - i_p) = \cos i_p$$

thus $n = \tan i_p$

(d) (i) $r = 90 - i_p = \underline{34^\circ}$

(ii) $n = \frac{\sin i}{\sin r} = \underline{1.48}$

7 (a) (i) $n = \tan i_p$
 $i_p = \tan^{-1} 1.33$
 $= \underline{53^\circ}$

(ii) angle of refraction = $90^\circ - 53^\circ = \underline{37^\circ}$

(b) (i) The path of the light is reversed for the same polarising condition:

$$i_p = \underline{37^\circ}$$

Alternatively $n_{\text{water} \rightarrow \text{air}} = \frac{1}{n_{\text{air} \rightarrow \text{water}}} = \frac{1}{1.33}$

$$i_p = \tan^{-1} \frac{1}{1.33} = 37^\circ$$

(ii) The refracted ray in air will be $\underline{53^\circ}$

Coulomb's inverse square law and electric field strength

$$1. \quad F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}$$

where F - force between the two charges (N), Q_1 and Q_2 - charges (C)
 ϵ_0 - permittivity of free space ($F\ m^{-1}$), r - separation of charges (m)

$$2. \quad F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} \quad F = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(1.5 \times 10^{-9})^2}$$

$F = 1.0 \times 10^{-10}$ N repulsion [the direction must be stated]

$$3. \quad 14 = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{r^2} \quad \text{giving } r = 4.1 \times 10^{-15} \text{ m}$$

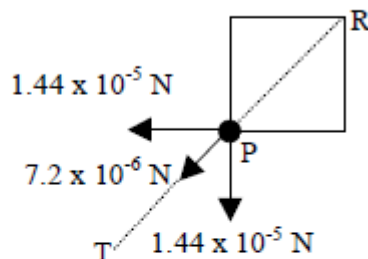
$$4. \quad \text{(a) Magnitude of } F \text{ due to charge } Q = 9 \times 10^9 \times \frac{4 \times 10^{-9} \times 4 \times 10^{-9}}{0.1^2}$$

$$= 1.44 \times 10^{-5} \text{ N}$$

Magnitude of F due to charge $S = 1.44 \times 10^{-5}$ N (same separation)

Magnitude of F due to charge $R = 7.2 \times 10^{-6}$ N ($r = 0.1414$ m)

Total force on charge is determined by vector addition.



Combining the two 1.44×10^{-5} N forces gives a force of 2.04×10^{-5} N in the same direction as the 7.2×10^{-6} N force.

$$\text{Hence total force on } P \text{ is } = (2.04 + 0.72) \times 10^{-5} \text{ N}$$

$$F = 2.8 \times 10^{-5} \text{ N}$$

in the direction RPT, shown on the diagram.

- (b) zero. The two 4.0 nC charges at opposite ends of a diagonal will exert an equal and opposite force on the -1.0 nC charge at the centre, hence the resultant force will be zero.

$$5. \quad F = 9 \times 10^9 \times \frac{20 \times 10^{-9} \times 20 \times 10^{-9}}{0.8^2} + 9 \times 10^9 \times \frac{20 \times 10^{-9} \times 20 \times 10^{-9}}{1.6^2}$$

$$F = 7.0 \times 10^{-6} \text{ N to the right}$$

6. The electric field strength is the electrostatic force on one coulomb of charge placed at that point.

$$7. \quad (a) \quad E = \frac{Q}{4\pi\epsilon_0 r^2} \quad (b) \quad E = \frac{V}{d}$$

8. See Student Material page 5.

$$9. \quad E = \frac{Q}{4\pi\epsilon_0 r^2} = 9 \times 10^9 \times \frac{2 \times 1.6 \times 10^{-19}}{(1 \times 10^{-10})^2}$$

$$= 2.88 \times 10^{11} \text{ V m}^{-1} \text{ (or N C}^{-1}\text{) away from the nucleus}$$

$$10. \quad E = \frac{Q}{4\pi\epsilon_0 r^2}; \quad 72\,000 = 9 \times 10^9 \times \frac{2 \times 10^{-6}}{r^2} \quad r = 0.50 \text{ m}$$

11. (a) Towards the bottom plate, perpendicular to the plates.

$$(b) \quad (i) \quad E = \frac{V}{d} = \frac{4.0 \times 10^3}{0.02} = 2.0 \times 10^5 \text{ V m}^{-1}$$

(ii) The same $E = 2.0 \times 10^5 \text{ V m}^{-1}$, the field is uniform between the plates.

Coulomb's Inverse Square Law

Note: the following examples use $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$

1 (a) Use Coulomb's Law
$$F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

$$= 9.0 \times 10^9 \times \frac{2 \times 10^{-8} \times 4 \times 10^{-8}}{(2 \times 10^{-2})^2}$$

$$= \underline{0.018 \text{ N}}$$

This is a force of attraction since the charges have opposite sign.

(b) $F = 1.0 \times 10^{-4} \text{ N}$
$$r^2 = \frac{1}{4\pi\epsilon_0} \times \frac{Q_1 Q_2}{F}$$

$$= 9.0 \times 10^9 \times \frac{(2 \times 10^{-8} \times 4 \times 10^{-8})}{1.0 \times 10^{-4}}$$

thus $r = \underline{0.27 \text{ m}}$

2
$$F_g = \frac{GM_1 M_2}{r^2} \quad F_e = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$$

We could work out the forces separately. However it is easier to simply take the ratio $\frac{F_e}{F_g}$. Then the r^2 will cancel.

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0} \times Q_1 Q_2}{G M_1 M_2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6.67 \times 10^{-11} \times 1.673 \times 10^{-27} \times 9.11 \times 10^{-31}}$$

$$= \underline{2.3 \times 10^{39}}$$

Thus the electrostatic force is almost 10^{40} times bigger than the gravitational force between sub-atomic particles. We can therefore safely neglect gravitational effects for such particles.

3 Here $F_e = F_g$

thus
$$\frac{1}{4\pi\epsilon_0} \frac{Q Q}{r^2} = \frac{G M m}{r^2} \quad r^2 \text{ cancels}$$

$$9 \times 10^9 \times Q^2 = 6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}$$

$$Q^2 = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 7.3 \times 10^{22}}{9 \times 10^9}$$

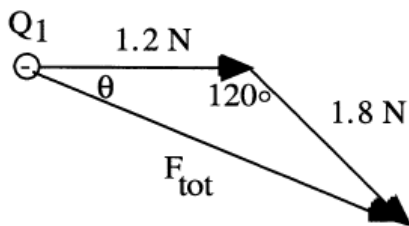
and $Q = \underline{5.7 \times 10^{13} \text{ C}}$

This is a huge charge and as you will see later it is not possible to create a positive charge in isolation. Such a possibility for the Earth could not arise.

4

$$\begin{aligned}
 F_2 &= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2} \\
 &= 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 3 \times 10^{-6}}{(0.15)^2} \\
 &= 1.2 \text{ N in a direction from } Q_1 \text{ towards } Q_2 \\
 F_3 &= 9 \times 10^9 \times \frac{1 \times 10^{-6} \times 2 \times 10^{-6}}{(0.10)^2} \\
 &= 1.8 \text{ N in a direction away from } Q_1, \text{ along the line } Q_3 Q_1
 \end{aligned}$$

The resultant force is the vector sum of these two forces.



Use the cosine rule:

$$\begin{aligned}
 F_{\text{tot}}^2 &= 1.2^2 + 1.8^2 - 2 \times 1.2 \times 1.8 \cos 120^\circ \\
 &= 6.84
 \end{aligned}$$

$$\text{thus } F_{\text{tot}} = \sqrt{6.84} = \underline{2.6 \text{ N}}$$

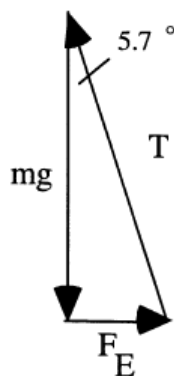
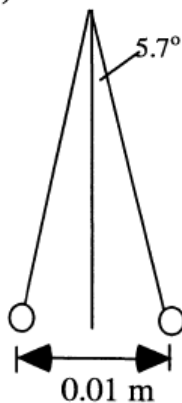
To find the direction, θ , use the sine rule:

$$\begin{aligned}
 \frac{\sin \theta}{1.8} &= \frac{\sin 120^\circ}{2.6} \\
 \theta &= \underline{37^\circ}
 \end{aligned}$$

The resultant force is 2.6 N at 37° as shown in the sketch opposite.

This problem could have been done by scale drawing.

5 (a)



$$\begin{aligned}
 F_E &= mg \tan 5.7^\circ \\
 &= 0.10 \times 10^{-3} \times 9.8 \times \tan 5.7 \\
 &= 9.78 \times 10^{-5} \\
 &= \underline{9.8 \times 10^{-5} \text{ N}} \quad (2 \text{ sig. figs.})
 \end{aligned}$$

Alternatively use the components of the tension T.

$$\begin{aligned}
 (b) \quad F_E &= \frac{1}{4\pi\epsilon_0} \frac{Q Q}{r^2} \quad \text{thus } Q^2 = \frac{9.8 \times 10^{-5} \times (0.01)^2}{9 \times 10^9} \\
 Q &= \underline{1.04 \times 10^{-9} \text{ C}}
 \end{aligned}$$

$$(c) \quad Q = I t \quad \text{thus } t = \frac{Q}{I}$$

$$t = \frac{1.04 \times 10^{-9}}{1 \times 10^{-11}} = \underline{104 \text{ s}}$$

(d) Charge one of the spheres by touching it to the dome of a Van de Graaff generator. Now touch the uncharged sphere to the charged sphere. If both spheres are the same size, the charge will be shared equally.

6

<p>(a)</p>	<p>(b)</p> $\tan \theta = \frac{0.016}{0.80} = 0.02$ $F_E = mg \tan \theta$ $= 0.50 \times 10^{-3} \times 9.8 \times 0.02$ $= \underline{9.8 \times 10^{-5} \text{ N}}$
------------	---

7

--	--

The charged plastic rod causes the polar molecules in the paper to line up in the way shown. Note that the paper is overall neutral. The paper is attracted because the positive charge is closer to the negative rod.

Electric fields and electrostatic potential

- (a) zero
 (b) $E = \frac{Q}{4\pi\epsilon_0 r^2} = 9 \times 10^9 \times \frac{30 \times 10^{-6}}{0.04^2} = 1.7 \times 10^8 \text{ V m}^{-1}$ away from the sphere.
 (c) $E = 2.7 \times 10^5 \text{ V m}^{-1}$ [as above with $r = 1 \text{ m}$] away from the sphere.
- See Student Material page 6. Notice that the object charged by induction, the electroscope, is not touched by the charging object, the negatively charged rod.
- The electrostatic potential at a point is the work done to bring one coulomb of charge from infinity to that point.

$$4. \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

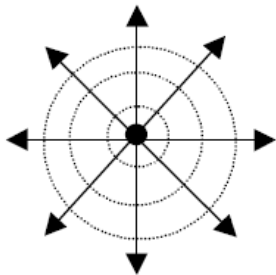
$$5. \quad V = \frac{Q}{4\pi\epsilon_0 r} = 9 \times 10^9 \times \frac{4 \times 10^{-9}}{3} = 12 \text{ V}$$

$$6. \quad V_A = 9 \times 10^9 \times \frac{6 \times 10^{-9}}{2} = 27 \text{ V} \quad \text{and} \quad V_B = 9 \times 10^9 \times \frac{-6 \times 10^{-9}}{5} = -10.8 \text{ V}$$

Potential difference $V_{AB} = 16.2 \text{ V}$ where A is negative compared to B

- A surface on which the potential is the same at all points. No work would be done to move a charge between two points on the surface.

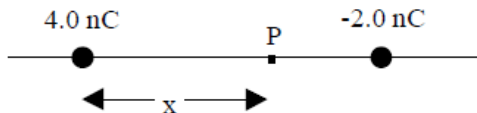
8.



The field lines are full lines and the lines of equipotential are broken lines.

Notice that the field lines and lines of potential are perpendicular to each other.

9.



For zero potential at point P: $V(\text{due to } 4.0 \text{ nC}) + V(\text{due to } -2.0 \text{ nC}) = 0$

$$9 \times 10^9 \times \frac{4 \times 10^{-9}}{x} + 9 \times 10^9 \times \frac{-2 \times 10^{-9}}{(0.12 - x)} = 0$$

$$4(0.12 - x) - 2x = 0 \quad \text{and} \quad 0.48 - 4x - 2x = 0 \quad \text{giving } x = 0.08 \text{ m}$$

10. Electrostatic force, electric field strength.

$$11. (a) V_X = 9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.1} + 9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.412} = 280 \text{ V}$$

$$V_Y = 2 \times (9 \times 10^9 \times \frac{2.5 \times 10^{-9}}{0.224}) = 201 \text{ V}$$

$$(b) V_{XY} = 79 \text{ V}$$

$$12. \text{Energy} = VQ \quad 10^{-14} = V \times 1.6 \times 10^{-19} \quad \text{giving } V = 62.5 \text{ kV}$$

Electric Field Strength

$$1 \quad E = \frac{F}{Q} = \frac{0.02}{4.0 \times 10^{-6}} \\ = \underline{5000 \text{ N C}^{-1}}$$

$$2 \text{ (a)} \quad E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \\ 1.0 = 9 \times 10^9 \times \frac{Q}{(1.0)^2}$$

$$\text{thus} \quad Q = \frac{1}{9 \times 10^9} = \underline{1.1 \times 10^{-10} \text{ C}}$$

$$(b) \quad E \propto \frac{1}{d^2} \quad \text{the distance has been doubled thus field strength quarters.} \\ E \text{ at } 2.0 \text{ m will be } \underline{0.25 \text{ N C}^{-1}}$$

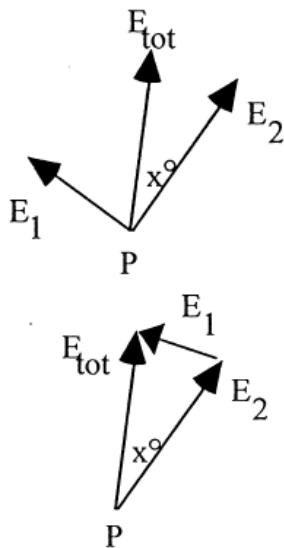
$$3 \text{ (a)} \quad E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

$$\alpha\text{-particle has 2 protons} \quad Q = 2 \times 1.6 \times 10^{-19} \text{ C}$$

$$\text{thus} \quad E = 9 \times 10^9 \times \frac{3.2 \times 10^{-19}}{(5 \times 10^{-3})^2} \\ = \underline{1.2 \times 10^{-4} \text{ N C}^{-1}}$$

$$(b) \text{ for one proton the charge is halved compared to part (a) and } E \propto Q \\ \text{thus } E \text{ will also be halved: } E = \underline{6.0 \times 10^{-5} \text{ N C}^{-1}}$$

4 (a)



The angle at P is a right angle.

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r^2} = 9 \times 10^9 \times \frac{18.8 \times 10^{-9}}{(0.12)^2} \\ = 1.2 \times 10^4 \text{ N C}^{-1} \text{ (2 sig figs)}$$

in direction shown, from 18.8 nC charge

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r^2} = 9 \times 10^9 \times \frac{10 \times 10^{-9}}{(0.05)^2} \\ = 3.6 \times 10^4 \text{ N C}^{-1}$$

in direction shown, from 10 nC charge

$$E_{\text{tot}} = \sqrt{E_1^2 + E_2^2} \text{ (magnitude)} \\ = \underline{3.8 \times 10^4 \text{ N C}^{-1}}$$

(b) From the sketch above:

$$\tan x^\circ = \frac{E_1}{E_2} = \frac{1.2 \times 10^4}{3.6 \times 10^4} \\ x = \underline{18^\circ}$$

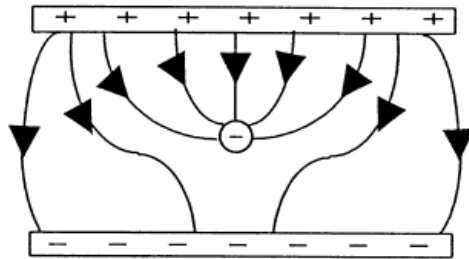
- 5 (a) For a stationary charged sphere:
the downward force of gravity = upward electric force
thus $mg = EQ$

$$E = \frac{mg}{Q}$$

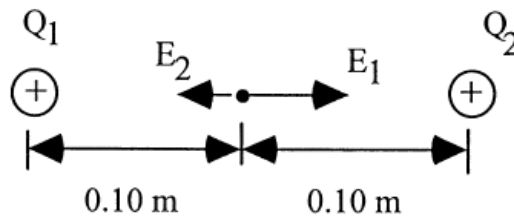
$$E = \frac{2.0 \times 10^{-5} \times 9.8}{5.0 \times 10^{-9}}$$

$$= \underline{3.9 \times 10^4} \text{ N C}^{-1}$$

(b)



6 (a)



E_1 will be double the size of E_2 because Q_1 is double the size of Q_2 .

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1^2} = 9 \times 10^9 \times \frac{8.0 \times 10^{-9}}{(0.10)^2} = 7.2 \times 10^3 \text{ N C}^{-1}$$

thus $E_2 = 3.6 \times 10^3 \text{ N C}^{-1}$ Directions of E_1 and E_2 are shown in sketch

$E_{\text{tot}} = E_1 - E_2 = \underline{3.6 \times 10^3} \text{ N C}^{-1}$ in the direction from Q_1 to Q_2 .

(b) When $E_{\text{tot}} = 0 \text{ N C}^{-1}$ $E_1 = -E_2$ and in magnitude $E_1 = E_2$

$$\frac{1}{4\pi\epsilon_0} \frac{Q_1}{r_1^2} = \frac{1}{4\pi\epsilon_0} \frac{Q_2}{r_2^2} \quad \text{and} \quad r_2 = (0.20 - r_1)$$

$$\frac{Q_1}{r_1^2} = \frac{Q_2}{(0.20 - r_1)^2}$$

$$\text{thus} \quad \frac{Q_1}{Q_2} = \frac{r_1^2}{(0.20 - r_1)^2} \quad \text{but } Q_1 = 2 Q_2$$

$$\text{thus} \quad 2 = \frac{r_1^2}{(0.20 - r_1)^2} \quad \text{take square root of each side}$$

$$\sqrt{2} = \frac{r_1}{(0.20 - r_1)}$$

$$\text{thus} \quad r_1 = \sqrt{2} \times (0.20 - r_1) \quad \text{and} \quad 0.20 \times \sqrt{2} = (1 + \sqrt{2}) r_1$$

$$r_1 = \frac{0.20 \times \sqrt{2}}{(1 + \sqrt{2})} = \underline{0.12} \text{ m}$$

(c) (i) Use Coulomb's Law: $F = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r^2}$

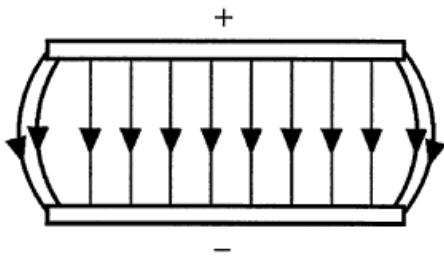
$$= 9 \times 10^9 \times \frac{8.0 \times 10^{-9} \times 4.0 \times 10^{-9}}{(0.2)^2}$$

$$= \underline{7.2 \times 10^{-6} \text{ N}}$$

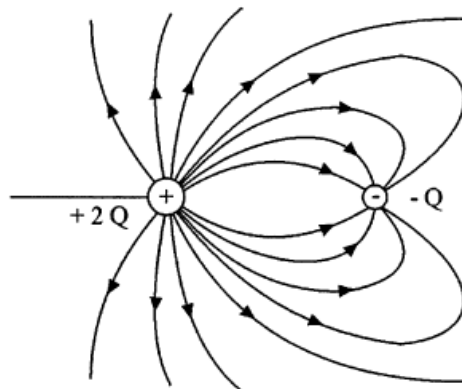
(ii) Using $F = mg$ $F = 5.0 \times 10^{-4} \times 9.8$

$$F = \underline{4.9 \times 10^{-3} \text{ N}}$$

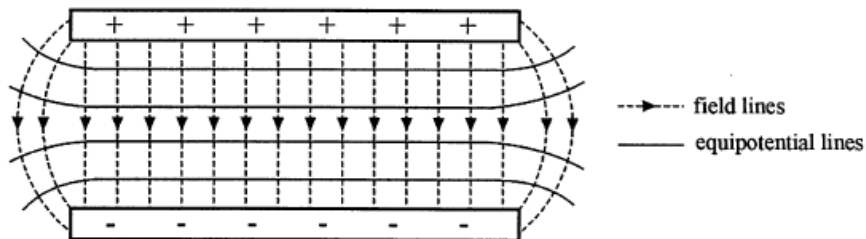
7 (a)



(b)



8



field lines and equipotential lines are always at right angles

Electrostatic Potential

$$1 \quad V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = 9 \times 10^9 \times \frac{3 \times 10^{-9}}{0.05} \\ = \underline{540 \text{ V}}$$

- 2 (a) Potential is a scalar - therefore there is no need to consider any directions.
Note that all the charges are equidistant from C at 0.10 m.
Potential due to a +1.0 nC charge at C:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} = 9 \times 10^9 \times \frac{1.0 \times 10^{-9}}{0.10} = 90 \text{ V}$$

Potential due to a negative charge is negative.

$$V_C = (2 \times 90) + (-2 \times 90) + (6 \times 90) + (3 \times 90) \\ = 180 - 180 + 540 + 270 \\ = \underline{810 \text{ V}}$$

- (b) Distance from +2.0 nC and +6.0 nC to D = 0.158 m (by Pythagoras)
Distance from -2.0 nC and +3.0 nC to D = 0.0707 m (by Pythagoras).

$$\text{thus } V \text{ due to } +6.0 \text{ nC at D} = 9 \times 10^9 \times \frac{6.0 \times 10^{-9}}{0.158} = 341.8 \text{ V}$$

$$V \text{ due to } +2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.158} = 113.9 \text{ V}$$

$$V \text{ due to } -2.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -254.6 \text{ V}$$

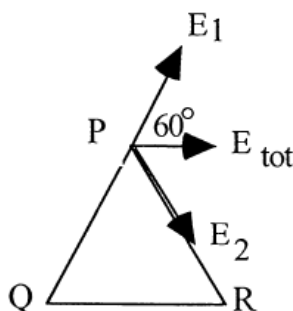
$$V \text{ due to } +3.0 \text{ nC at D} = 9 \times 10^9 \times \frac{2.0 \times 10^{-9}}{0.0707} = -381.9 \text{ V}$$

$$V_{\text{tot}} = 583 \text{ V at D}$$

thus potential difference between C and D = $810 - 583 = \underline{+227 \text{ V}}$

$$3 \quad \text{Energy} = QV = Q \times \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \\ = 1.6 \times 10^{-19} \times 9 \times 10^9 \times \frac{1.6 \times 10^{-19}}{5 \times 10^{-11}} \\ = \underline{4.6 \times 10^{-18} \text{ J}}$$

- 4 (a)



$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = 9 \times 10^9 \times \frac{1 \times 10^{-8}}{(2.0 \times 10^{-2})^2}$$

$$= 2.25 \times 10^5 \text{ N C}^{-1} \text{ in direction away from Q}$$

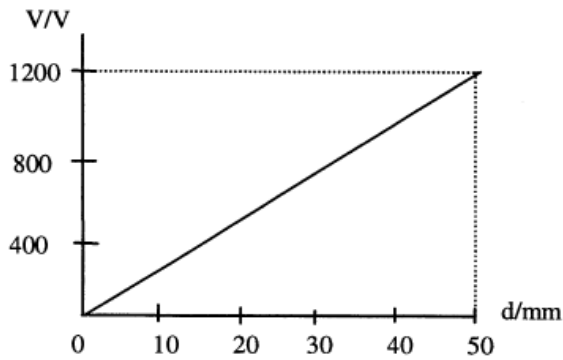
$$E_2 = 2.25 \times 10^5 \text{ N C}^{-1} \text{ from P towards R}$$

$$E_{\text{tot}} = E_1 \cos 60^\circ + E_2 \cos 60^\circ$$

$$= \underline{2.25 \times 10^5 \text{ N C}^{-1}} \text{ in the direction shown}$$

- (b) The potential at P will be zero because charges at Q and R are equal in size and opposite in sign and both points are equidistant from P.

5 (a)



$$(b) E = \frac{V}{d} = \frac{1200}{5 \times 10^{-2}}$$

$$= \underline{2.4 \times 10^4} \text{ V m}^{-1}$$

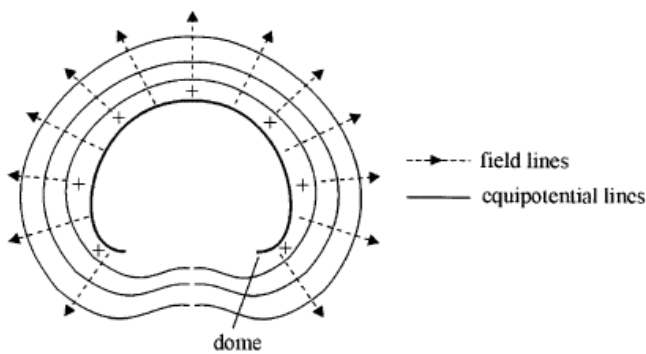
Direction of field is toward the lower plate

(c) Take the gradient of the graph

$$E = -\frac{dV}{dx}$$

6 (a) Equipotential surfaces have the same potential at all points. Note that moving a charge between two points on an equipotential surface needs no work.

(b) In the sketch below the solid lines show the electric field and the dotted lines show the equipotential surface lines.



7

$$E = \frac{V}{d} = \frac{1500}{0.02}$$

$$= \underline{7.5 \times 10^4} \text{ V m}^{-1}$$

8 (a)

$$E = \frac{V}{d} = \frac{2000}{0.15}$$

$$= \underline{1.33 \times 10^4} \text{ V m}^{-1}$$

(b) (i) Energy change: electrical potential energy into kinetic energy

(ii) work done = QV

$$= 1.6 \times 10^{-19} \times 2000$$

$$= \underline{3.2 \times 10^{-16} \text{ J}}$$

(iii) $QV = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2QV}{m}}$$

$$= \underline{2.7 \times 10^7} \text{ m s}^{-1}$$

9 (a) The proton will have a uniform acceleration of $1.27 \times 10^{-12} \text{ m s}^{-2}$ towards the negative plate, assuming air resistance is negligible. Any downward force due to gravity has been ignored.

(b) (i) The work done will be the same as in the previous question because the charge on the proton is the same as the charge on the electron.

(ii) Using $QV = \frac{1}{2} m v^2$

$$v = \sqrt{\frac{2QV}{m}}$$

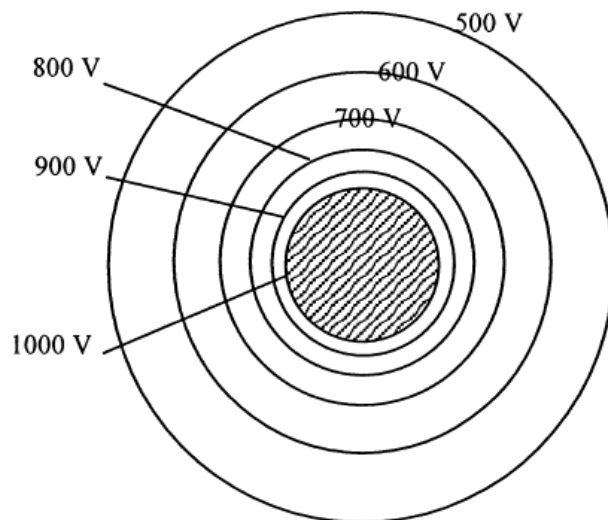
$$= \underline{6.2 \times 10^5} \text{ m s}^{-1}$$

10 (a) $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$ thus $V \propto \frac{1}{r}$ or $V_1 r_1 = V_2 r_2$

$$\text{thus } r_2 = \frac{V_1 r_1}{V_2} = \frac{0.05 \times 1000}{900} = 0.056 \text{ m}$$

$$r_3 = \frac{0.05 \times 1000}{800} = 0.063 \text{ m}$$

similarly, $r_4 = 0.071 \text{ m}$; $r_5 = 0.083 \text{ m}$; $r_6 = 0.10 \text{ m}$



Notice that equipotential lines which are separated by the same amount of p.d., in this case 100 V, become further apart as the radius increases.

(b) $V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$

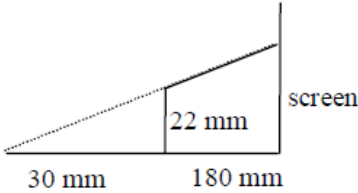
$$\text{thus } 1000 = 9 \times 10^9 \times \frac{Q}{0.05}$$

$$Q = \frac{1000 \times 0.05}{9 \times 10^9} = \underline{5.6 \times 10^{-9}} \text{ C}$$

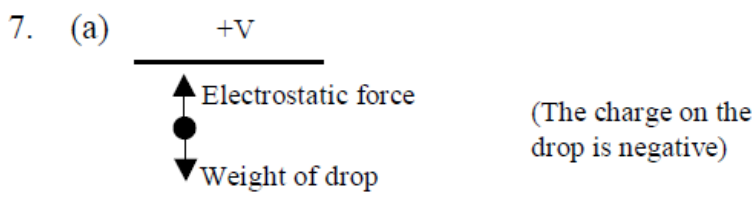
11 (a) $v = \sqrt{\frac{2eV}{m}} = \underline{5.9 \times 10^8} \text{ m s}^{-1}$

(b) This calculated velocity is greater than the speed of light which is not physically possible. A relativistic calculation is needed, an example is shown for interest at the end of the Mechanics unit solutions.

Charges in motion

- Work = p.d. x charge = $1000 \times 6.0 \times 10^{-6} = 6.0 \text{ mJ}$
 - Electrostatic potential energy is transformed into kinetic energy as the charge is accelerated towards the top plate.
- $V = Ed$ $3.0 \times 10^4 = 5.0 \times 10^5 \times d$ giving $d = 0.060 \text{ m}$ or 60 mm
 - The electric field strength will double.
 - change in $E_k =$ change in electrical energy
 $\frac{1}{2} mv^2 - 0 = VQ$ where V is the p.d.
 $v = \sqrt{\frac{2Ve}{m}}$ where e is the charge, and m the mass, of an electron.
- In the horizontal direction: velocity of electron entering the plates
 $\frac{1}{2} mv^2 = 2.88 \times 10^{-16}$ thus $v = 2.51 \times 10^7 \text{ m s}^{-1}$.
 - In the horizontal direction:
 Time taken to travel pass plates at 0.15 m long = $5.98 \times 10^{-9} \text{ s}$
 In the vertical direction: time = $5.98 \times 10^{-9} \text{ s}$ initial velocity = 0
 Force on electron due to electric field = $EQ = 1.4 \times 10^4 \times 1.6 \times 10^{-19} \text{ N}$
 Acceleration = $\frac{F}{m} = \frac{1.4 \times 10^4 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 2.46 \times 10^{15} \text{ m s}^{-2}$
 Using $s = ut + \frac{1}{2} at^2$ deflection = 0.044 m
 - The electron will travel in a straight line. There is no unbalanced force on the electron because there is no electric field outside the plates.
- $\frac{1}{2} mv^2 = Ve$ $\frac{1}{2} \times 9.11 \times 10^{-31} \times v^2 = 200 \times 1.6 \times 10^{-19}$
 thus $v = 8.38 \times 10^6 \text{ m s}^{-1}$
 - In the horizontal direction, time taken = $\frac{\text{length of plates}}{\text{horizontal speed}} = 3.58 \times 10^{-9} \text{ s}$
 In the vertical direction, $a = \frac{F}{m} = \frac{1.0 \times 10^3 \times 1.6 \times 10^{-19}}{50 \times 10^{-3} \times 9.11 \times 10^{-31}} = 3.51 \times 10^{15} \text{ m s}^{-2}$
 Initial velocity vertically = 0 hence using $s = ut + \frac{1}{2} at^2$ gives $s = 0.022 \text{ m}$
 - 

The electron travels in a straight line after leaving the plates.
 By proportion $\frac{\text{deflection}}{22} = \frac{210}{30}$
 Giving the deflection on the screen = 154 mm
- $1.6 \times 10^{-19} \times 125 \times 10^3 = \frac{1}{2} \times 9.11 \times 10^{-31} \times v^2$ gives $v = 2.09 \times 10^8 \text{ m s}^{-1}$
 - This speed is more than 60% of the speed of light. Relativistic effects must be considered when speeds are greater than 10 % of the speed of light.
- Millikan determined the charge on a number of small charged drops. He noticed that the charges were all multiples of a certain smallest charge, $1.6 \times 10^{-19} \text{ C}$. This suggested that it was not possible to obtain a charge with a fraction of this value.



(b) weight down = electrostatic force upwards

$$mg = EQ = \frac{VQ}{d}$$

$$0.01 \times 10^{-9} \times 9.8 = \frac{V \times 5 \times 1.6 \times 10^{-19}}{20 \times 10^{-3}}$$

$$V = 2.45 \times 10^6 \text{ V}$$

(c) The drop would accelerate upwards, since the electrostatic force is now greater than the weight of the drop. The drop will accelerate in the direction of the resultant force.

8. (a) Excess charge on each drop is calculated using $Q = \frac{mgd}{V}$ and $gd = 0.392$

Mass of drop/ 10^{-15} kg	2.6	1.2	1.6	2.3	4.8	5.9	1.8	3.7
p.d. / V	1592	2940	1960	2818	2940	14455	1470	4533

Charge/ 10^{-19} C	6.4	1.6	3.2	3.2	6.4	1.6	4.8	3.2
No. excess electrons	4	1	2	2	4	1	3	2

(b) These are all whole numbers. No fractional charges were found.

9. (a) Change in $E_k = \frac{1}{2} mv^2 - 0$ with zero E_k at the distance of closest approach.

(b) Change in electrostatic $E_p = \frac{qQ}{4\pi\epsilon_0 r} - 0$ [potential energy = $V \times$ charge]

where q and Q are the charges on the alpha particle and oxygen atom.

(c) At closest approach change in $E_k =$ change in E_p

$$\frac{1}{2} mv^2 - 0 = \frac{qQ}{4\pi\epsilon_0 r} - 0$$

$$\text{rearranging gives } r = \frac{2qQ}{4\pi\epsilon_0 mv^2}$$

$$(d) r = 9 \times 10^9 \times \frac{2 \times (2 \times 1.6 \times 10^{-19}) \times (8 \times 1.6 \times 10^{-19})}{6.7 \times 10^{-27} \times (1.9 \times 10^6)^2} = 3.0 \times 10^{-13} \text{ m}$$

$$10. \text{ From above } v^2 = \frac{2qQ}{4\pi\epsilon_0 mr} = 9 \times 10^9 \times \frac{2 \times (2 \times 1.6 \times 10^{-19}) \times (26 \times 1.6 \times 10^{-19})}{6.7 \times 10^{-27} \times 1.65 \times 10^{-13}}$$

$$\text{and } v = 4.7 \times 10^6 \text{ m s}^{-1}$$

Charges in Motion

1 (a)

$$F = E Q$$

$$= 1.2 \times 10^6 \times 1.6 \times 10^{-19}$$

$$a = \frac{F}{m} = \frac{1.2 \times 10^6 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}} = 2.11 \times 10^{17}$$

$$= \underline{2.1 \times 10^{17} \text{ m s}^{-2}}$$

(b) (i) Using $v = u + at$ $u = 0$ gives $t = \frac{v}{a}$

$$t = \frac{3.0 \times 10^7}{2.11 \times 10^{17}} = \underline{1.42 \times 10^{-10} \text{ s}}$$

(ii) Using $s = ut + \frac{1}{2} a t^2$

$$= 0 + \frac{1}{2} \times 2.11 \times 10^{17} \times (1.42 \times 10^{-10})^2$$

$$= 2.1 \times 10^{-3} \text{ m} = \underline{2.1 \text{ mm}}$$

2 (a) The oil drop must be **negatively** charged.

(b) Calculate the size of the electric field: $E = \frac{V}{d} = \frac{2000}{0.02} = 1.0 \times 10^5 \text{ N C}^{-1}$

At balance:

$$F_{\text{elect.}} = F_{\text{grav.}}$$

$$E Q = m g$$

$$Q = \frac{m g}{E} = \frac{4.9 \times 10^{-15} \times 9.8}{1.0 \times 10^5}$$

$$= \underline{4.8 \times 10^{-19} \text{ C}}$$

This is equivalent to an **excess** of three electrons on the oil drop.

3 (a) Use the principle of conservation of energy:

thus work done on a charge by the electric field = E_k 'lost'

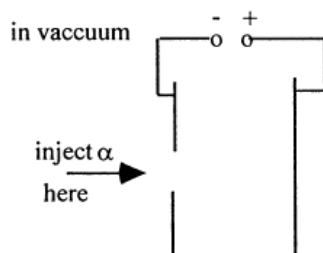
$$F \times d = \frac{1}{2} m v^2 \quad \text{also: } F = E Q: \quad E Q d = \frac{1}{2} m v^2$$

$$\text{thus } E = \frac{\frac{1}{2} m v^2}{Q d} = \frac{\frac{1}{2} \times 6.7 \times 10^{-27} \times (5 \times 10^6)^2}{3.2 \times 10^{-19} \times 6.0 \times 10^{-2}}$$

$$E = \underline{4.4 \times 10^6 \text{ N C}^{-1}}$$

(Alternatively use: $v^2 = u^2 + 2as$, $F_{\text{un}} = ma$ and $E = \frac{F_{\text{un}}}{m}$)

(b)



(c) Gamma rays cannot be stopped by an electric field because gamma rays do not have any electric charge. Gamma rays are electromagnetic radiation of very high frequency.

4 (a)
$$t = \frac{d}{v} = \frac{5.0 \times 10^{-2}}{6.0 \times 10^6}$$

$$= \underline{8.33 \times 10^{-9}} \text{ s (keeping 3 sig figs for parts (b) to (d))}$$

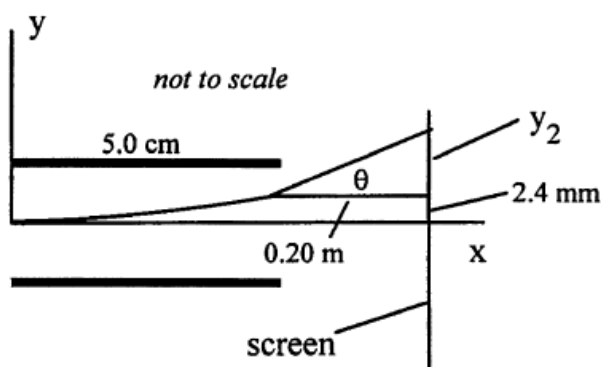
(b) Let the displacement in the vertical plane be y .

$$y = u_y t + \frac{1}{2} a t^2 \quad \text{where} \quad a = \frac{F}{m} = \frac{EQ}{m}$$

$$u_y = 0 \quad \text{thus} \quad y = \frac{1}{2} \times \frac{4 \times 10^2 \times 1.6 \times 10^{-19} \times (8.33 \times 10^{-9})^2}{9.11 \times 10^{-31}} = \underline{2.4 \times 10^{-3}} \text{ m}$$

Thus the vertical displacement experienced by an electron is 2.4 mm.

(c)



θ can be worked out from the combination of vertical and horizontal velocities at the plate edge.

$$\text{(From above } a = \frac{EQ}{m} = 7.025 \times 10^{13} \text{ m s}^{-2}\text{)}$$

$$v_{\text{vert}} = a t = 7.025 \times 10^{13} \times 8.33 \times 10^{-9}$$

$$= 5.85 \times 10^5 \text{ m s}^{-1}$$

$$v_{\text{hor}} = 6.0 \times 10^6 \text{ m s}^{-1}$$

$$\tan \theta = \frac{v_{\text{vert}}}{v_{\text{hor}}} = \frac{5.85 \times 10^5}{6.0 \times 10^6}$$

$$\theta = \underline{5.6^\circ}$$

(d) Also $\tan \theta = \frac{y_2}{0.20}$ thus $y_2 = \tan \theta \times 0.20$

$$y_2 = 1.96 \times 10^{-2} \text{ m} = 19.6 \text{ mm}$$

thus total vertical displacement at screen = 2.4 mm + 19.6 mm = 22.0 mm

thus $y_{\text{tot}} = \underline{22 \text{ mm}}$ (2 sig figs)

5
$$QV = \frac{1}{2} m v^2$$

giving
$$v = \sqrt{\frac{2QV}{m}}$$

$$= \underline{5.13 \times 10^8} \text{ m s}^{-1}$$

This speed is greater than the speed of light. Hence this speed is not possible.

A relativistic calculation is needed which gives $v = 2.7 \times 10^8 \text{ m s}^{-1}$. An example is shown, for interest, at the end of the solutions to the Mechanics unit. Such calculations are **not** required for examination purposes.

6 (a) Head on collisions of α -particles with gold nuclei happen very rarely because the nucleus of the gold atom is so very small compared to the overall size of the atom. Most of the atom is empty space.

(b) At closest approach $E_p = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r}$ $q =$ electronic charge and
 $Q =$ charge of gold nucleus

$$E_k \text{ of } \alpha\text{-particle} = \frac{1}{2} m v^2$$

Change of $E_k =$ change of E_p where r is the closest distance of approach.

$$\frac{1}{2} m v^2 - 0 = \frac{1}{4\pi\epsilon_0} \frac{qQ}{r} - 0 \quad (\text{initial } E_p = 0 \text{ and final } E_k = 0)$$

thus $r = \frac{1}{4\pi\epsilon_0} \frac{2qQ}{mv^2}$

$$= 9 \times 10^9 \times \frac{2 \times 3.2 \times 10^{-19} \times 79 \times 1.6 \times 10^{-19}}{6.7 \times 10^{-27} \times (2.0 \times 10^7)^2}$$

$$= \underline{\underline{2.7 \times 10^{-14} \text{ m}}}$$

7 (a) $m = \rho V = 870 \times \frac{4}{3} \pi \times (1.62 \times 10^{-6})^3$
 $= \underline{\underline{1.55 \times 10^{-14} \text{ kg}}}$

(b) At balance $F_{\text{elect}} = F_{\text{grav}}$
 $E Q = m g$
 thus $Q = \frac{m g}{E} = \frac{1.55 \times 10^{-14} \times 9.8}{1.9 \times 10^5}$
 $= \underline{\underline{8.0 \times 10^{-19} \text{ C}}}$

(c) $e =$ one electronic charge $= 1.6 \times 10^{-19} \text{ C}$
 thus no. of charges $= \frac{Q}{e} = 5$ a whole number.

8 $\frac{1}{2} m v^2 = Q V$
 $v^2 = \frac{2QV}{m}$
 $v = \sqrt{2 \frac{Q}{m} V}$
 $= \sqrt{2 \times 1.8 \times 10^{11} \times 250}$
 $= \underline{\underline{9.5 \times 10^6 \text{ m s}^{-1}}}$

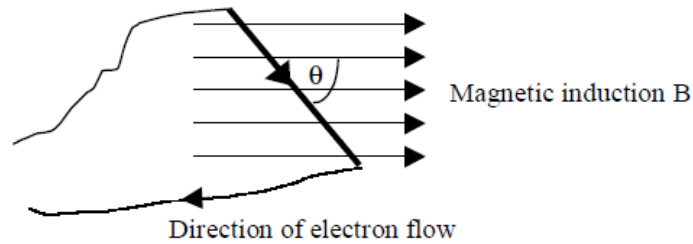
Notice that this speed is much less than the speed of light.

Electromagnetism

- A magnetic field is produced around moving charges.
 - The charge must be moving across the magnetic field, or the magnetic field must be changing relative to the position of the charge.
 - One tesla is the magnetic induction of a magnetic field in which a conductor of length one metre, carrying a current of one ampere perpendicular to the field, is acted on by a force of one newton.

2. (a) $F = I/B\sin\theta$

- (b) For a current in the direction shown, the force on the wire is directed into the page.



(c) $F = I/B\sin\theta$ giving $9.5 \times 10^{-3} = 2 \times 25 \times 10^{-3} \times 0.70 \times \sin\theta$
 $\theta = 16^\circ$

3. $F = I/B\sin\theta$ $0.20 = 10 \times 0.8 \times B$ ($\sin\theta = 1$ since θ is 90°)
 $B = 25 \text{ mT}$ (0.025 T)

4. (a) $F = I/B\sin\theta$ $0.30 = I \times 0.50 \times 0.10$ for θ at 90° and $\sin\theta = 1$
 $I = 6 \text{ A}$

- (b) If θ is less than 90° , $\sin\theta$ will be less than 1, and a larger current I will be required.

5. (a) $F = I/B\sin\theta$ $F = 7.0 \times 200 \times 10^{-3} \times 0.15 \times \sin 35$ $F = 0.12 \text{ N}$
 (b) Out of the page, perpendicular to both the magnetic field and the wire.

6. Magnetic induction $B = \frac{\mu_0 I}{2\pi r}$

7. See Student Material page 20

B is magnetic induction (T)

μ_0 is the permeability of free space (H m^{-1})

I_1 and I_2 are the currents in the conductors (A) r is the distance between them (m)

F/l is the force per unit length (N m^{-1})

8. $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$ giving $8.89 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times 2.0 \times I_2}{2\pi \times 90 \times 10^{-3}}$ $I_2 = 2.0 \text{ A}$

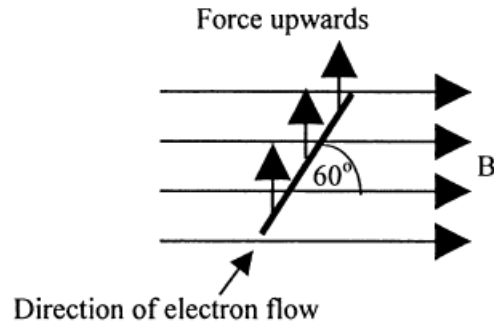
9. Weight per m length = magnetic force per m length
 0.075 = $\frac{4\pi \times 10^{-7} \times I^2}{2\pi \times 5.0 \times 10^{-3}}$
 Current in each wire = 43 A

Force on a Conductor

1 (a)

$$\begin{aligned}
 F &= I l B \sin \theta \\
 &= 7.5 \times 0.05 \times 0.04 \times \sin 60^\circ \\
 &= \underline{0.013 \text{ N}}
 \end{aligned}$$

(b)



(c) For maximum force, $\theta = 90^\circ$ and all the conductor, 50 mm, is in the field.

2

$$\begin{aligned}
 F &= I l B \sin \theta \\
 4.5 \times 10^{-3} &= 1.4 \times 50 \times 10^{-3} \times 0.09 \times \sin \theta \\
 \theta &= \underline{46^\circ}
 \end{aligned}$$

3 (a) To remove tension in the supporting leads the magnetic force has to be equal and opposite to the weight of the wire.

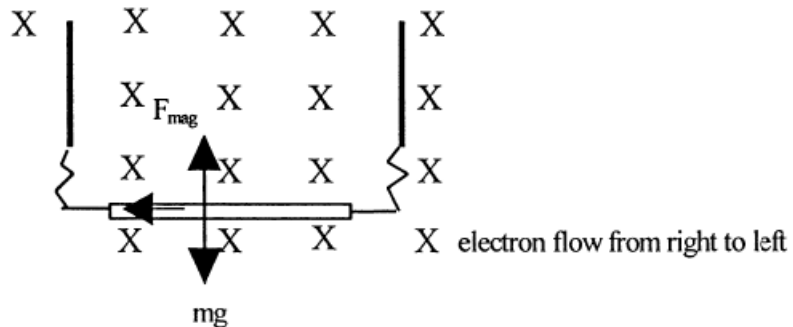
$$\begin{aligned}
 W &= mg = 0.025 \times 9.8 \\
 &= 0.245 \text{ N}
 \end{aligned}$$

using $F = I l B \sin \theta$ and $F = 0.245 \text{ N}$ and $\theta = 90^\circ$

$$I = \frac{F}{l B \sin \theta} = \frac{0.245}{0.75 \times 0.50 \times 1}$$

$$I = \underline{0.65 \text{ A}}$$

(b) Apply Right Hand Rule



4 (a)

$$F = I l B \sin \theta$$

For $\theta = 90^\circ$ the 0.25 m sides are perpendicular to the field

$$F = 0.25 \times 0.25 \times 0.40 \times 1$$

$$F = \underline{0.025 \text{ N}}$$

(b)

$$T = F r \quad \text{for each force} \quad T = 0.025 \times 0.075$$

Total torque = $2 \times 0.025 \times 0.075 = 3.75 \times 10^{-3} \text{ N m}$ (for each turn of wire)

Thus for the whole coil: Torque = $120 \times 3.75 \times 10^{-3}$
= $\underline{0.45 \text{ N m}}$

(c)

As the coil rotates the 0.25 m sides of the coil make angles less than 90° with the field. The force on the wire decreases so the torque decreases. When the coil is perpendicular to the field these sides are momentarily parallel to the field and the torque will be zero.

5 (a)

Since the balance reading is less, this suggests that there is an **upward** force on the magnet assembly exerted by the current in the wire.

$$\text{difference in readings} = 95.6 \text{ g} - 93.2 \text{ g} = 2.4 \text{ g}$$

$$F = 2.4 \times 10^{-3} \times 9.8 = 0.02352$$

$$= \underline{0.024 \text{ N}}$$

(b)

$$F = I l B \sin \theta \quad (\theta = 90^\circ)$$

$$0.024 = 4.0 \times 0.06 \times B \times 1$$

$$B = \underline{0.1 \text{ T}}$$

(c)

Reversing the direction of the current in the wire reverses the direction of the force. This direction is now downward and will **increase** the reading by 2.4 g.

$$\text{new reading on balance} = 95.6 + 2.4 = \underline{98.0 \text{ g}}$$

(d)

When north faces north, the field is zero between the magnets (in centre). There will be no magnetic force on the wire. The balance will read 95.6 g.

6 (a)

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

thus

$$F = \frac{4\pi \times 10^{-7} \times 16 \times (2500)^2}{2\pi \times 0.20}$$

$$= \underline{100 \text{ N}}$$

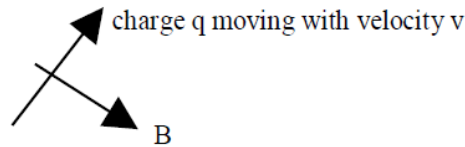
(b)

If the wires were suspended freely they would attract each other. If they touched there would be a short circuit which could start a fire.

Motion in a magnetic field

1.

Consider a charge q moving with a constant speed v **perpendicular** to a magnetic field of magnetic induction B .



We know that $F = I/B\sin\theta$.

Consider the charge q moving through a distance l . (The italic l is used to avoid confusion with the number one.)

Then time taken $t = \frac{l}{v}$ and current $I = \frac{q}{t} = \frac{qv}{l}$ giving $lI = qv$.

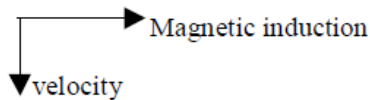
Substituting into $F = I/B\sin\theta$, with $\sin\theta = 1$ since $\theta = 90^\circ$, gives:

$$\boxed{F = qvB}$$

2. $F = qvB$ $F = 1.6 \times 10^{-19} \times 3 \times 10^4 \times 0.80 = 3.8 \times 10^{-15} \text{ N}$

3. (a) $F = qvB$ $F = 1.6 \times 10^{-19} \times 2.0 \times 10^5 \times 0.50 = 1.6 \times 10^{-14} \text{ N}$

(b) The force is perpendicular to both the velocity and the magnetic field.



For these directions of v and B the force is vertically into the page.

(c) Central force $F = \frac{mv^2}{r}$ giving $qvB = \frac{mv^2}{r}$ and $r = \frac{mv}{qB}$

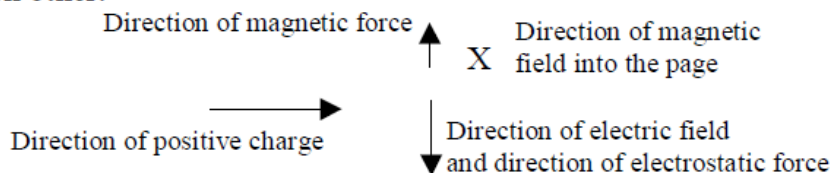
$$r = \frac{9.11 \times 10^{-31} \times 2 \times 10^5}{1.6 \times 10^{-19} \times 0.5} = 2.3 \times 10^{-6} \text{ m}$$

4. The direction of the velocity of the electron must make an angle with the direction of the magnetic field.

The component of velocity perpendicular to the field causes the electron to move in a circle.

The component of velocity parallel to the field causes the electron to move along the direction of the field.

5. (a) Electric and magnetic fields arranged so their forces on a charged particle oppose each other. The electric and magnetic field must be perpendicular to each other.

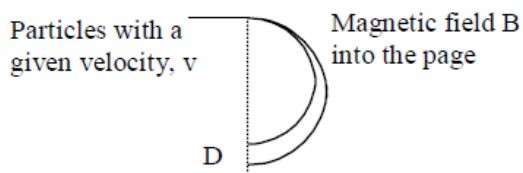


For the example shown above, a magnetic field is directed into the page and an electric field acts down the page. The magnetic force and the electrostatic force on a positive charge oppose each other. These fields are then said to be 'crossed'.

The fields are often adjusted such that the two forces are equal in magnitude, $F_e = F_m$. The velocity of the positive charge would then remain constant.

- (b) $F_e = F_m$ also $F_e = Eq$ and $F_m = qvB$ giving $Eq = qvB$ and $v = \frac{E}{B}$.
- (c) From the above equation the velocity does not depend on the mass or the charge of the particle but only on the values of E and B.
- (d) Charged particles enter the region with the magnetic field only, at right angles to the field. The particle will move in a circle with a radius given by

$$r = \frac{mv}{qB}$$



The particles all have the same velocity hence the radius will depend on the mass and charge of each particle. Different ions will meet a detector at D in different places.

6. (a) From the question above $v = \frac{E}{B} = \frac{V}{Bd}$
 (b) $qvB = \frac{mv^2}{r}$ or $evB = \frac{mv^2}{r}$ giving $\frac{e}{m} = \frac{v}{rB}$
 (c) $\frac{e}{m} = \frac{v}{rB} = \frac{V}{rB^2d}$

7. Velocity of each ion is given by $\frac{1}{2} mv^2 = qV$

giving $v^2 = \frac{2qV}{m}$ ----- {1}

radius of curved path $r = \frac{mv}{qB}$ ----- {2}

and $r^2 = \frac{m^2 v^2}{q^2 B^2}$ thus $r^2 = \frac{m^2}{q^2 B^2} \left(\frac{2qV}{m} \right)$ substituting for v^2

giving $r = \sqrt{\frac{2Vm}{qB^2}}$

The difference in the two radii $r_{13} - r_{12} = \sqrt{\frac{2V}{qB^2}} (\sqrt{m_{13}} - \sqrt{m_{12}})$
 $= 5 \times 10^{-3} \text{ m}$

Hence $d = 0.01 \text{ m}$ (Because of the subtraction above the answer is to one significant figure only.)

[Alternatively the velocity of each ion can be calculated using {1} above to give v_{13} and v_{12} . Then equation {2} used to give the difference in the radii $r_{13} - r_{12} = \frac{1}{qB} (m_{13}v_{13} - m_{12}v_{12})$. This involves more calculations and potential loss in accuracy.]

Charged Particles in Magnetic Fields

$$1 \quad F = q v B \sin\theta$$

thus
$$B = \frac{F}{q v \sin\theta} = \frac{3.0 \times 10^{-17}}{1.6 \times 10^{-19} \times 1.0 \times 10^7 \times \sin 45^\circ}$$

$$= \underline{2.7 \times 10^{-5} \text{ T}}$$

$$2 \quad F_m = q v B \sin\theta \quad \text{and } \theta = 90^\circ$$

$$= 1.6 \times 10^{-19} \times 2.8 \times 10^8 \times 3.3 \times 10^{-5} \times 1.0$$

$$= 1.478 \times 10^{-15} \text{ N}$$

$$F_g = m g \quad \text{notice the speed of proton is } > 10\% \text{ speed of light}$$

Use
$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$
 to find the relativistic mass m

$$m = 4.660 \times 10^{-27}$$

$$F_g = 4.660 \times 10^{-27} \times 9.8 = 4.567 \times 10^{-26} \text{ N}$$

thus
$$\frac{F_m}{F_g} = \frac{1.478 \times 10^{-15}}{4.567 \times 10^{-26}} = \underline{3.2 \times 10^{10}}$$

If the relativistic mass is not used an answer of 9.0×10^{-10} is obtained!

$$3 \quad F = q v B = \frac{m v^2}{r} \quad \text{giving } r = \frac{m v}{q B}$$

$$\frac{1}{2} m v^2 = 4.2 \times 10^{-12} \quad \text{to find } v \text{ from value given for } E_k.$$

$$v = \sqrt{\frac{2 \times 4.2 \times 10^{-12}}{1.673 \times 10^{-27}}} = 7.086 \times 10^7 \text{ m s}^{-1}$$

from above
$$r = \frac{m v}{q B} = \frac{1.673 \times 10^{-27} \times 7.086 \times 10^7}{1.6 \times 10^{-19} \times 0.28}$$

$$= \underline{2.6 \text{ m}}$$

$$4 \quad \text{from } qvB = \frac{mv^2}{r} \quad r = \frac{m v}{q B} \quad \text{and } v = r\omega = r \times 2\pi f$$

giving
$$f = \frac{qB}{2\pi m}$$
 use this to compare frequency for α and electron

$$\frac{f_\alpha}{f_e} = \frac{q_\alpha B}{2\pi m_\alpha} \times \frac{2\pi m_e}{q_e B} = \frac{q_\alpha m_e}{q_e m_\alpha} = \frac{2m_e}{m_\alpha} \quad \text{since } q_\alpha = 2q_e$$

$$= \frac{2 \times 9.11 \times 10^{-31}}{6.68 \times 10^{-27}} = \underline{2.73 \times 10^{-4}}$$

Alternatively: $f_e = 3.67 \times 10^3 f_\alpha$

5 (a) The magnetic force supplies the central acceleration.

$$qvB = \frac{mv^2}{r} \quad \text{giving } r = \frac{m v}{q B} \quad \text{also } v = r\omega \quad \text{and } \omega = 2\pi f$$

hence
$$\frac{v}{r} = 2\pi f \quad \text{and } \frac{v}{r} = \frac{qB}{m} \quad \text{giving } f = \frac{qB}{2\pi m}$$

(b) From the equation in (a) above: $B = \frac{2\pi mf}{q}$

$$B = \frac{2\pi \times 3.34 \times 10^{-27} \times 1.2 \times 10^7}{1.6 \times 10^{-19}} = 1.574$$

$$= \underline{1.6} \text{ T}$$

(c) At maximum radius R: $v = \frac{qBR}{m}$

E_k of deuterons emerging: $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \times \left[\frac{qBR}{m} \right]^2$

$$E_k = \frac{q^2 B^2 R^2}{2m}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (1.574)^2 \times (0.50)^2}{2 \times 3.34 \times 10^{-27}}$$

$$= \underline{2.4 \times 10^{-12} \text{ J}}$$

6 (a) If undeflected:

$$F_{\text{mag}} = F_{\text{elect}}$$

$$q v B = q E$$

$$v = \frac{E}{B}$$

(b) $v = \frac{1.4 \times 10^5}{0.70}$

$$= \underline{2.0 \times 10^5} \text{ m s}^{-1}$$

(c) (i) $r = \frac{m v}{q B}$ thus $m = \frac{q r B}{v}$

$$m = \frac{1.6 \times 10^{-19} \times 0.07 \times 0.7}{2.0 \times 10^5}$$

$$= \underline{3.9 \times 10^{-26} \text{ kg}}$$

(ii) The ions of the different isotopes will have different radii. They will therefore show up at different points on the photographic record. The less massive particles will have a larger radius.

7 (a) $v = \frac{q r B}{m}$ from $qvB = \frac{mv^2}{r}$

$$= \frac{3.2 \times 10^{-19} \times 0.45 \times 1.2}{6.68 \times 10^{-27}} = 2.59 \times 10^7$$

$$= \underline{2.6 \times 10^7} \text{ m s}^{-1}$$

(b) $v = \frac{2\pi r}{T}$

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 0.45}{2.59 \times 10^7}$$

$$= \underline{1.1 \times 10^{-7} \text{ s}}$$

(c) $E_k = \frac{1}{2} m v^2 = \frac{1}{2} \times 6.68 \times 10^{-27} \times (2.6 \times 10^7)^2$

$$= \underline{2.3 \times 10^{-12} \text{ J}}$$

- 8 (a) Particle X is moving in a direction parallel to the magnetic field. This means that it will not experience a magnetic force. Particle X will therefore carry on in a straight line with no change of speed.

Particle Y follows a circular path because it enters the magnetic field at right angles to the field direction.

Particle Z follows a helical (spiral) path because it enters the magnetic field at an angle.

$$(b) \quad r = \frac{m v}{q B} \quad \text{from } qvB = \frac{mv^2}{r}$$

$$r = \frac{2 \times 10^{-6} \times 1.673 \times 10^{-27}}{1.6 \times 10^{-19} \times 1.3 \times 10^{-5}}$$

$$= 1.61 \times 10^3 \text{ m}$$

$$= \underline{1.6 \text{ km}}$$

- 9 (a) (i)

$$F_{\text{mag}} = F_{\text{elect}}$$

$$q v B = q E$$

thus $v = \frac{E}{B} \quad \text{and} \quad E = \frac{V}{d}$

$$v = \frac{V}{Bd}$$

- (ii)

$$B = \frac{9 \times 10^{-7} \text{ NI}}{a}$$

$$= \frac{9 \times 10^{-7} \times 320 \times 0.31}{0.073} = 1.22 \times 10^{-3}$$

$$= \underline{1.2 \times 10^{-3} \text{ T}}$$

- (iii)

thus

$$v = \frac{V}{Bd} = \frac{1200}{1.22 \times 10^{-3} \times 0.045} = 2.19 \times 10^7$$

$$= \underline{2.2 \times 10^7 \text{ m s}^{-1}}$$

- (b) (i)

$$r = \frac{m v}{q B} \quad \text{from } qvB = \frac{mv^2}{r}$$

and

$$\frac{e}{m} = \frac{v}{rB} \quad \text{since here } q = e$$

- (ii)

$$r = \frac{L^2 + y^2}{2y}$$

$$r = \frac{0.055^2 + 0.015^2}{2 \times 0.015}$$

$$= 0.108 \text{ m}$$

$$\frac{e}{m} = \frac{v}{rB} = \frac{2.19 \times 10^7}{0.108 \times 1.2 \times 10^{-3}}$$

$$= \underline{1.7 \times 10^{11} \text{ C kg}^{-1}}$$

The accepted value for $\frac{e}{m}$ is $1.76 \times 10^{11} \text{ C kg}^{-1}$

10 (a) $E = \frac{V}{d} = \frac{1200}{0.05} = \underline{2.40 \times 10^4} \text{ V m}^{-1}$

(b) (i) $B = \frac{8\mu_0 NI}{\sqrt{125} r}$
 $= \frac{8 \times 4\pi \times 10^{-7} \times 320 \times 0.25}{11.2 \times 0.068} = 1.058 \times 10^{-3}$
 $= \underline{1.06 \times 10^{-3} \text{ T}}$

(ii) $v = \frac{E}{B} = \frac{2.40 \times 10^4}{1.058 \times 10^{-3}}$
 $v = \underline{2.27 \times 10^7} \text{ m s}^{-1}$ (keeping 3 sig figs)

(c) (i) time taken to cross between plates $t = \frac{L}{v}$

deflection, $y = \frac{1}{2} a t^2 = \frac{1}{2} a \frac{L^2}{v^2}$

(ii) thus $y = \frac{1}{2} \times \frac{eE}{m} \times \frac{L^2}{v^2}$ since $a = \frac{F}{m} = \frac{eE}{m}$
 $\frac{e}{m} = \frac{2yv^2}{EL^2} = \frac{2 \times 0.01 \times (2.27 \times 10^7)^2}{2.40 \times 10^4 \times 0.05^2}$
 $= \underline{1.72 \times 10^{11} \text{ C kg}^{-1}}$

11 (a) Electric potential energy = kinetic energy gained

$$eV = \frac{1}{2} m v^2 \quad \text{and } V = 1000 \text{ V}$$

thus $\frac{e}{m} = \frac{v^2}{2 \times 1000}$

(b) (i) $B = \frac{9 \times 10^{-7} NI}{r}$
 $= \frac{9 \times 10^{-7} \times 320 \times 0.26}{0.068}$
 $= \underline{1.10 \times 10^{-3} \text{ T}}$

(ii) $v = \frac{E}{B} = \frac{2.0 \times 10^4}{1.10 \times 10^{-3}}$ since $E = \frac{1000}{0.05} = 2 \times 10^4$
 $= \underline{1.82 \times 10^7} \text{ m s}^{-1}$

(c) $\frac{e}{m} = \frac{v^2}{2 \times 1000} = \frac{(1.82 \times 10^7)^2}{2000}$
 $= \underline{1.66 \times 10^{11} \text{ C kg}^{-1}}$

(d) Accepted value for $\frac{e}{m} = 1.76 \times 10^{11} \text{ C kg}^{-1}$

(i) An uncertainty of 5% in the value in (c) above:

gives $\frac{e}{m} = (1.66 \pm 0.08) \times 10^{11} \text{ C kg}^{-1}$

(ii) The calculated uncertainty does not bring the measured value within range of the accepted value. The measured value is about 6% too low.

Capacitors in d.c. circuits

Numerical answers:

1. (a) $5.0 \times 10^{-3} \text{ C}$
(b) (i) 1.25 A
(ii) current decreases exponentially
2. 0.5 F
3. $2.25 \times 10^{-5} \text{ C}$
4. (a) 1.0 F
(b) 0.8 C
5. (b) 4.9 mF
6. (e) (i) 40 A
(ii) $4.0 \times 10^2 \text{ C}$ or $4.0 \times 10^{-4} \text{ C}$
7. (e) 12 V
10. 8 ms
12. (b) (i) 8 s
(ii) 7.3 V

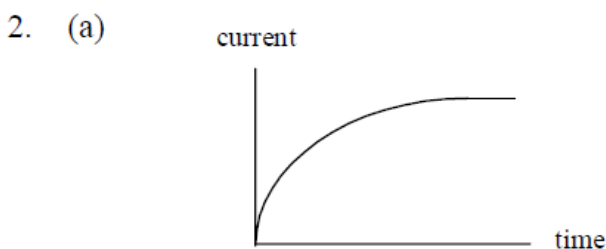
Capacitors in a.c. circuits

Numerical answers:

2. (a) $3.2 \times 10^3 \mu$
(b) 1.6 mA
3. (a) 500μ
(b) $6.4 \mu\text{F}$
4. (a) 0.24 A r.m.s
(b) $4.8 \mu\text{F}$
5. (a) $600 \mu\text{F}$
(b) 18.9 mA
(c) 5.66 V
6. (a) 4μ
(b) $2.7 \times 10^{-4} \text{ F}$ (actual values 4μ and $3.0 \times 10^{-4} \text{ F}$)

Self-inductance

- (a) A magnet is moved in and out a coil. The coil is connected to a voltmeter and a deflection is observed when the magnet is moving relative to the coil, see Electrical Phenomena - Student Material page 27.
 - (b) Increase the relative speed of the magnet and coil, increase the magnetic induction of the magnet, increase the number of turns on the coil.



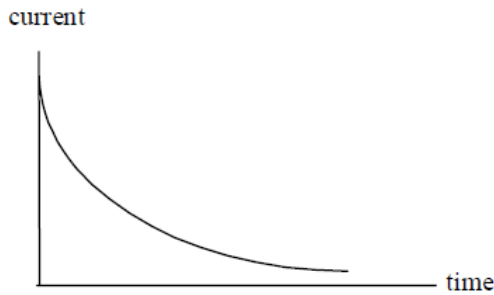
- (b) When the switch is closed the current increases from zero. The magnetic field through the inductor will increase. An e.m.f. is induced across the inductor due to the changing magnetic field through the inductor. This induced e.m.f. acts against the current preventing the current reaching its maximum value immediately.
 - (c) The current would reach its steady value quicker, see Electrical Phenomena – Student Material page 28.
- When a steady current is passed through a coil a constant magnetic field is established through the coil. When the current through the coil changes, the magnetic field through the coil will change. A changing magnetic field will cause an induced e.m.f. through the coil.
[Note: The induced e.m.f. will act in a direction to oppose the change causing it. Thus the induced e.m.f. produced when the current increases will act in a direction as to oppose the increase. It will act **against** the current direction.]
 - (a) When the current decreases the magnetic field will decrease and an e.m.f. will be induced.
 - (b) The induced e.m.f. will act in the **same** direction as the current, that is it will try to keep the current steady and stop the change in the magnetic field. The energy needed to do this comes from the energy which was stored in the magnetic field. When the magnetic field decreases this energy is released and to conserve energy work has to be done.
 - (a) $\mathcal{E} = -L \frac{dI}{dt}$
 - (b) The unit of inductance is henry (H)
 - (c) Energy = $\frac{1}{2} LI^2$

6. (a) As the current increases the magnetic field through the inductor increases. An e.m.f. is induced against the direction of the current. Thus the current takes time to reach its maximum value.
- (b) Using $V = IR$ steady current = $12/15 = 0.8$ A
- (c) Energy = $\frac{1}{2} LI^2 = \frac{1}{2} \times 0.40 \times 0.8^2 = 0.13$ J

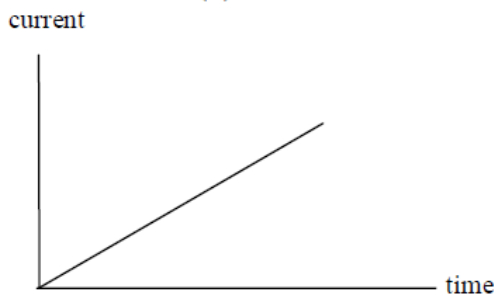
7. (a) p.d. = $I R = 0.1 \times 20 = 2 \text{ V}$
 (b) induced e.m.f. = $8 - 2 = 6 \text{ V}$
 (c) $\mathcal{E} = -L \frac{dI}{dt}$ $6 = -L \times (-100)$ $L = 0.06 \text{ H}$
 (d) Energy = $\frac{1}{2} LI^2 = \frac{1}{2} \times 0.06 \times 0.10^2 = 0.30 \text{ mJ}$

8. None

9. (a) (i)



(ii)



- (b) The reactance of the inductor is the opposition of the inductor to the alternating current. It is given by reactance $X_L = \frac{V}{I}$.
11. (a) When a switch is opened in a circuit containing an inductor the current will fall rapidly to zero. There will be a large change in the magnetic field through the inductor and this will cause a large induced e.m.f. at the switch terminals.



The inductor has a large opposition to a.c. signals. For d.c. signals the only opposition is the resistance of the coil. Assume that the resistance of the resistor is much larger than the resistance of the inductor. The p.d. across the inductor will be due to the a.c. signals and the p.d. across the resistor will be due to the d.c. signals. The inductor blocks the a.c.

Self-Inductance

1 (a) V across $R = IR = 8.0 \times 1.0 = 8.0 \text{ V}$
 thus e.m.f. across inductor $= 12 - 8.0 = \underline{4.0 \text{ V}}$

(b) $\epsilon = -L \frac{dI}{dt}$
 $-4.0 = -L \times 400$ (4.0 V is a back e.m.f.)
 $L = \frac{4.0}{400} = \underline{0.01 \text{ H}}$

(c) When the switch is closed and current is zero, all of the e.m.f. will be across the inductor: $\epsilon = 12 \text{ V}$.

$$\frac{dI}{dt} = \frac{\epsilon}{L} = \frac{12}{0.01} = \underline{1200 \text{ A s}^{-1}}$$

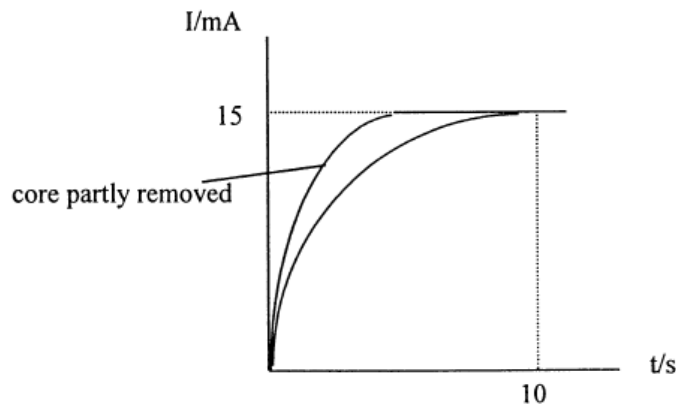
(d) $I_{\text{final}} = \frac{\epsilon}{R} = \frac{12}{1.0} = \underline{12 \text{ A}}$

(e) $E = \frac{1}{2} L I^2$
 $= \frac{1}{2} \times 0.01 \times 12^2$
 $= \underline{0.72 \text{ J}}$

2 (a) (i) When the switch is closed the current starts to increase in the circuit. This *changing* current produces a *changing* magnetic field which in turn induces an e.m.f. across the coil. This e.m.f. opposes the build up of the current (Lenz's law). This is observed as a delay in the current reaching a final steady value.

(ii) When the current reaches its final steady value there is no induced e.m.f. across the inductor and therefore no back e.m.f. generated. The resistance in the circuit and the e.m.f. of the supply determine this steady current.

(b) The removal of the soft iron core reduces the inductance of the coil. This will result in a faster build up of current because the back e.m.f. will be less.



(c) $R_{\text{coil}} = \frac{\text{e.m.f.}}{I_{\text{final}}} = \frac{3.0}{0.015} = \underline{200 \Omega}$

- 3 (a) (i) p.d. across R at start is 0 V
 (ii) initial current is also zero.
 (iii) initial induced e.m.f across L is the e.m.f. of the supply, 10 V
 (iv) $E = \frac{1}{2} L I^2$ but $I = 0$ A thus $E = \underline{0}$ J

(b) (i) $V = I R = 0.04 \times 40 = \underline{1.6}$ V

(ii) thus $\epsilon = 10 - 1.6 = 8.4$ V

(iii) $\epsilon = -L \frac{dI}{dt}$

$$-8.4 = -2.0 \times \frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{8.4}{2.0} = \underline{4.2} \text{ A s}^{-1}$$

(iv) $E = \frac{1}{2} L I^2$
 $= \frac{1}{2} \times 2.0 \times 0.04^2$
 $= \underline{1.6 \times 10^{-3}} \text{ J}$

- 4 (a) The self-inductance of a coil is given by $\epsilon = -L \frac{dI}{dt}$. The inductance is one henry if an e.m.f. of one volt is induced when the current changes at a rate of one ampere per second.

- (b) (i) Lamp X lights more slowly due to the self-inductance of the inductor L. When the circuit is switched on the current grows and produces a changing magnetic field in the inductor. This in turn generates an e.m.f. which by Lenz's law opposes the original current. There is no such effect with a resistor, hence lamp Y lights immediately.

- (ii) When the switch is closed $\epsilon = 10$ V

$$\epsilon = -L \frac{dI}{dt} \quad \text{and} \quad L = \frac{10}{0.50} = \underline{20} \text{ H}$$

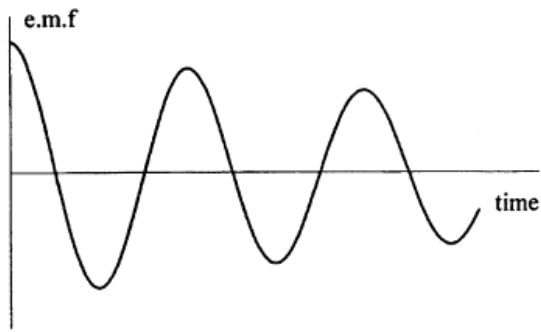
- (iii) Current in inductor branch = $\frac{P}{V} = \frac{3}{6} = 0.50$ A

p.d. across L = $10 - 6.0 = 4.0$ V

thus $R_L = \frac{V}{I} = \frac{4.0}{0.50} = \underline{8.0} \Omega$ using $V = IR$

- 5 (a) When S is opened the current in the primary collapses. This produces a large change in magnetic field in the primary. This in turn produces a change in magnetic field in the linked secondary coil, which gives an large induced e.m.f. across the spark plug.
 (b) If there are more turns in the secondary (step-up), a larger e.m.f. will be produced across the spark plug.
 (c) The energy for the spark comes from the battery via the electromagnetic field set up in the coils and core.

6 (a)



The oscillations will be damped due to Lenz's law. The magnetic field in the coil will oppose the movement of the magnet.

- (b) When the magnet momentarily stops the induced e.m.f. is zero. Relative movement is needed to induce an e.m.f.
- (c) When magnet movement is reversed the induced e.m.f. will also be reversed.
- (d) The fastest movement results in the maximum induced e.m.f.

Inductors and a.c.

Numerical answers:

2. (a) 250Ω
(b) 0.12 A
3. (a) 500Ω
(b) 1.6 H
4. (a) 0.12 A
(b) 30 V
5. **Lamp Z:** no change in brightness when frequency is altered. The resistance of a resistor does not change with frequency.
Lamp Y: as frequency increases the lamp dims because there is a greater back e.m.f. generated at a higher frequency. The inductive reactance has increased, i.e. the opposition to a.c. increases.
Lamp X: as frequency increases the lamp becomes brighter because the capacitor allows a greater current to pass. The capacitive reactance has decreased, i.e. the opposition to a.c. decreases.
6. (a) Loudspeaker A will reproduce the high frequency signals while loudspeaker B will reproduce the low frequency signals.
- (b) Both high and low frequency signals have a choice of path at the top of the circuit, where the inductor L and capacitor C_2 join. Higher frequency signals will pass through the capacitor, C_2 , because there is less opposition (capacitive reactance) for that route. After passing loudspeaker A the high frequency signals take the low opposition route through capacitor C_1 .
- The low frequency signals will pass through the inductor because this route has a lower opposition (inductive reactance) for low frequencies. The low frequencies signals, after passing through inductor L , will pass through loudspeaker B rather than pass through C_1 . A capacitor has a larger opposition to low frequency signals.
7. (b) (i) $6.4 \times 10^{-6} \text{ F}$
(ii) 2.6 H (actual values $6.0 \times 10^{-6} \text{ F}$ and 2.6 H)

Electromagnetic radiation

Numerical answers:

4. (a) $4.0 \times 10^2 \text{ V m}^{-1}$
(b) $4.5 \times 10^{14} \text{ Hz}$
(c) (ii) $1.3 \times 10^{-6} \text{ T}$